

Contents

Chapter I

Representations of Points by Boundary Measures

- § 1. Distinguished Classes of Functions on a Compact Convex Set 1
- Classes of continuous and semicontinuous, affine and convex functions.—Uniform and pointwise approximation theorems.—Envelopes.—*Grothendieck's completeness theorem.—Theorems of Banach-Dieudonné and Krein-Šmulyan*
- § 2. Weak Integrals, Moments and Barycenters 9
- Preliminaries and notations from integration theory.—An existence theorem for weak integrals.—Vague density of point-measures with prescribed barycenter.—*Choquet's barycenter formula for affine Baire functions of first class, and a counterexample for affine functions of higher class*
- § 3. Comparison of Measures on a Compact Convex Set 21
- Ordering of measures.—The concept of dilation for simple measures.—The fundamental lemma on the existence of majorants.—Characterization of envelopes by integrals.—*Dilation of general measures.—Cartier's Theorem*
- § 4. Choquet's Theorem 31
- A characterization of extreme points by means of envelopes.—The concept of a boundary set.—Hervé's theorem on the existence of a strictly convex function on a metrizable compact convex set.—The concept of a boundary measure, and Mokobodzki's characterization of boundary measures.—The integral representation theorem of Choquet and Bishop-de Leeuw.—A maximum principle for superior limits of l.s.c. convex functions.—Bishop-de Leeuw's integral theorem relatively to a σ -field on the extreme boundary.—*A counterexample based on the "porcupine topology".*
- § 5. Abstract Boundaries Defined by Cones of Functions 44
- The concept of a Choquet boundary.—Bauer's maximum principle.—The Choquet-Edwards theorem that Choquet boundaries are Baire spaces.—The concept of a Šilov boundary.—Integral representation by means of measures on the Choquet boundary

§ 6. Unilateral Representation Theorems with Application to Simplicial Boundary Measures 55

Ordered convex compacts.—Existence of maximal extreme points.—Characterization of the set of maximal extreme points as a Choquet boundary.—Definition and basic properties of simplicial measures.—Existence of simplicial boundary measures, and the Caratheodory Theorem in \mathbb{R}^n .—Decomposition of representing boundary measures into simplicial components

Chapter II

Structure of Compact Convex Sets

§ 1. Order-unit and Base-norm Spaces 67

Basic properties of (Archimedean) order-unit spaces.—A representation theorem of Kadison.—The vector-lattice theorem of Stone-Kakutani-Krein-Yosida.—Duality of order-unit and base-norm spaces.

§ 2. Elementary Embedding Theorems 79

Representation of a closed subspace A of $C_{\mathbb{R}}(X)$ as an $A(K)$ -space by the canonical embedding of X in A^* .—The concept of an "abstract compact convex" and its regular embedding in a locally convex Hausdorff space.—The connection between compact convex sets and locally compact cones.

§ 3. Choquet Simplexes 84

Riesz' decomposition property and lattice cones.—Choquet's uniqueness theorem.—Choquet-Meyer's characterizations of simplexes by envelopes.—Edward's separation theorem.—Continuous affine extensions of functions defined on compact subsets of the extreme boundary of a simplex.—Affine Borel extensions of functions defined on the extreme boundary of a simplex.—*Examples of "non-metrizable" pathologies in simplexes.*

§ 4. Bauer Simplexes and the Dirichlet Problem of the Extreme Boundary 103

Bauer's characterizations of simplexes with closed extreme boundary.—The Dirichlet problem of the extreme boundary.—A criterion for the existence of continuous affine extensions of maps defined on extreme boundaries.

§ 5. Order Ideals, Faces, and Parts 109

Elementary properties of order ideals and faces.—Extension property and characteristic number.—Archimedean and strongly Archimedean ideals and faces.—Exposed and relatively exposed faces.—Specialization to simplexes.—The concept of a "part", and an inequality of Harnack type.—Characterization of the parts of a simplex in terms of representing measures.—*An example of an Archimedean face which is not strongly Archimedean.*

§ 6. Split-faces and Facial Topology 128

Definition and elementary properties of split faces.—Characterization of split faces by relativization of orthogonal measures.—An extension theorem for continuous affine functions defined on a split face.—The facial topology.—Specialization to simplexes.—*Near-lattice ideals, and primitive ideal space.—The connection between facial topology and hull kernel topology.—Compact convex sets with sufficiently many inner automorphisms.—A remark on the applications to C^* -algebras.*

§ 7. The Concept of Center for $A(K)$ 153

Extension of facially continuous functions.—The facial topology is Hausdorff for Bauer simplexes only.—The concept of center, and the connections with facially continuous functions and order-bounded operators.—Convex compact sets with trivial center.—*An example of a prime simplex.—Størmer's characterization of Bauer simplexes.*

§ 8. Existence and Uniqueness of Maximal Central Measures Representing Points of an Arbitrary Compact Convex Set . . . 171

The relation $x \phi y$, and the concept of a primary point.—A point x is primary iff the local center at x is trivial.—The concept of a central measure.—Existence and uniqueness of maximal central measures in a special case.—The "lifting" technique.—Wils' theorem on the existence and uniqueness of maximal central measures which are pseudo-carried by the set of primary points.

Appendix 189

References 193

Subject Index 209