## CONTENTS

Chapter 1 Fundamental Concepts				
1	Introduction, 1			
2	A Simple Physical Model, 2			
3	A Classical Approach, 2			
4	The Invariant Imbedding Approach, 6			
5	Some Comments, Criticisms, and Questions, 9			
6	A Minor Variant of the Model of Section 2, 9			
7	A Major Variant of the Model of Section 2, 10			
8	The Classical Approach Extended, 11			
9	The Invariant Imbedding Approach Extended, 13			
10	Some Comments on Possible Uses of the Reflection and			
	Transmission Functions, 15			
11	Summary, 16			
	Problems, 17			
	References, 20			
Chapter 2 Additional Illustrations of the Invariant Imbedding Method 2				
1	Introduction, 22			
2	A Non-Linear Problem, 23			
3	A Generalization of the Model, 24			
4	Invariant Imbedding Formulation of the Model in			
	Section 3, 25			
5	The Linear Problem Revisited, 27			
6	A Perturbation Approach, 28			
7	Some Remarks and Comments, 31			
8	The Riccati Transformation Method, 32			
9	Summary, 35			
	Problems, 35			
	References, 38			

xii CONTENTS

Cha	Chapter 3 Functional Equations and Related Matters	
1	Introduction, 39	
2	A Basic Problem, 39	
3	The Basic Functional Equations, 40	
4	Some Applications of the Results of Section 3, 41	
5	Differential Equations Via Functional Equations, 45	
6	Summary, 47	
·	Problems, 47	
	References, 53	
Cha	apter 4 Existence, Uniqueness, and Conservation Relations	54
1	Introduction, 54	
2	The "Physics" of the Conservative Case and Its	
	Generalizations, 55	
3	Another Derivation of the Reflection Function, 56	
4	Some Conservation Relations, 57	
5	Proof of Existence in the Conservative Case, 59	
6	The Nonconservative Case: The Dissipation Function, 60	
7	The Existence Proof, 63	
8	Summary, 64	
	Problems, 65	
	References, 66	
Cha	Chapter 5 Random Walk	
1	Introduction, 67	
2	A One-Dimensional Random Walk Process, 67	
3	A Classical Formulation, 68	
4	An Invariant Imbedding Formulation, 70	
5	Some Remarks Concerning Section 4, 71	
6	Sketch of Another Approach, 72	
7	Expected Sojourn, 73	
8	A "Many-State" Case—Invariant Imbedding Approach, 74	
9	Time Dependent Processes—Classical Approach, 76	
10	Time-dependent Processes—Invariant Imbedding	
	Approach, 77	
11	A Multistep Process—Classical Approach, 78	
12	A Multistep Process—Invariant Imbedding Approach, 79	
13	Some Remarks on an Extension to a Continuous Case, 81	
14	Some Remarks About Random Walk in Two Dimensions, 82	
15	Summary, 83	

CONTENTS xiii

CU	DITENTS	XII	
	Problems, 83 References, 87		
Ch	Chapter 6 Wave Propagation		
1	Introduction, 88		
2	The Concept of a Plane Wave, 89		
3	A Two Medium Problem, 89		
4	A Multimedium Problem, 90		
5	Resolution of the Multimedium Problem by "Wavelet		
	Counting", 91		
6	A Continuous Medium Problem, 94		
7	An Analytical Approach to the Continuous Medium Problem,	97	
8	The W.K.B. Method, 98		
9	The Bremmer Series, 100		
10	Another Imbedding, 103		
11	Summary, 105		
	Problems, 105		
	References, 107		
Cha	apter 7 Time-Dependent Problems	108	
1	Introduction, 108		
2	A Time-Dependent Transport Problem—Particle-Counting		
_	Approach, 109		
3	Time-Dependent Transport by Transform Techniques, 114		
4	A Critique of the Foregoing, 116		
5	Time-Dependent Input, 118		
6	The Time-Dependent Wave Equation, 120		
7	The Diffusion Equation, 121		
8	Some Comments on the Previous Section, 124		
9	A Critique of Sections 7 and 8, 125		
10	Another Diffusion Problem, 126		
11	A Final Diffusion Problem, 127		
12	Summary, 129		
	Problems, 130		
	References, 132		
Cha	apter 8 The Calculation of Eigenvalues for Sturm-Liouville Type		
	Systems	133	
1	Introduction, 133		
2	Eigenlengths for Transport-like Equations in		
_	One Dimension, 134		
	One Dimension, 137		

xiv CONTENTS

3 4 5 6 7 8	The Calculation of Eigenlengths, 134 Some Generalizations, 137 Results for Sturm-Liouville Systems, 138 Connection with the Prüfer Transformation, 140 Some Numerical Examples, 142 Summary, 144 Problems, 145 References, 146	
Cha	pter 9 Schrödinger-Like Equations	147
1 2 3 4 5 6 7 8 9 10 11 11	Introduction, 147 Formulation of the Phase Shift Problem, 148 A Representation of the Solution for Large $t$ , 149 Partial Differential Equations for $a$ and $\psi$ , 151 Solution of the Partial Differential Equations for $a$ and $\psi$ , 153 Remarks on the Phase Shift Problem, 154 Formulation of the Eigenvalue Problem, 155 A Partial Differential Equation for $\tilde{b}$ and Its "Solution", 158 Resolution of the Difficulties, 161 Some Numerical Examples, 164 Some Remarks on the Eigenvalue Problem, 165 Summary, 166 Problems, 167 References, 168	
Cha	upter 10 Applications to Equations with Periodic Coefficients	169
1	Introduction, 169	
2	Statement of the Problem, 170	
3	The Differential Equations of Invariant Imbedding Over One Period, 171	
4	Difference Equations Over an Integral Number of Periods, 171	
5	Difference Equations Over a Nonintegral Number of Periods, 176	
6	The "Backwards" Equations, 177	
7	Some Numerical Results, 178	
8	The Method of Doubling, 180	
9	Trigonometry Revisited, 181	
10	Summary, 182	

CONTENTS xv

	Problems, 183 References, 185	
	References, 165	
Chapter 11 Transport Theory and Radiative Transfer		186
1	Introduction, 186	
2	The Linearized Boltzmann Equation, 187	
3	Some Remarks on Sections 1 and 2, 190	
4	Boundary and Initial Conditions, 192	
5	The Special Case of Slab Geometry and One Speed, 193	
6	The Time-Independent Slab Problem via Invariant	
	Imbedding—The Perturbation Approach, 196	
7	The Time-Independent Slab Problem Via Invariant	
	Imbedding—The Riccati Transformation, 202	
8	A Return to the Case of the Semi-Infinite Half-Space, 206	
9	Invariant Imbedding as a Calculational Device for Transport	
	Problems in a Slab, 208	
10	Transport Theory in Other Geometries, 209	
11	Time-Dependent Transport in a Slab Geometry, 210	
12	Summary, 214	
	Problems, 215	
	References, 217	
Cha	Chapter 12 Integral Equations	
1	Introduction, 219	
2	An Integral Equation for Transport in a Slab, 220	
3	A Pseudo-Transport Problem and Its Associated Integral	
	Equation, 223	
4	Representations for $\phi$ and $n$ , 226	
5	Derivation of the Principal Results, 227	
6	A Special Case, 232	
7	A Numerical Example and Some Remarks About	
	Eigenvalues, 234	
8	Further Remarks About the Foregoing, 238	
9	A Completely Different Approach, 239	
10	Summary, 242	
	Problems, 242	
	References, 245	
Author Index		247
Subject Index		249