Contents

Series Editor's Preface ix

Preface xi

Preface to the English Edition xiii

Basic Notation xv

Chapter 1. Preliminaries 1
1. Metric Spaces. Normed Spaces 1
2. Algebras and σ-Algebras of Sets 2
3. Countably Additive Functions and Measures 3
4. Measurable Functions 7
5. Integration 9
6. Function Spaces 14

Chapter 2. Hilbert Space Geometry. Continuous Linear Operators 18
1. Hilbert Space. The Space L_2 18
2. Orthonormal Systems 22
3. Projection Theorem. Orthogonal Expansions and Orthogonal Sums 26
4. Linear Functionals and Sesqui-linear Forms. Weak Convergence 32
5. The Algebra of Continuous Operators on H 36
6. Compact Operators 39
7. Bounded Self-adjoint Operators 41
8. Orthogonal Projections 44
9. Examples of Hilbert Spaces and Orthonormal Systems 47
10. Examples of Continuous Functionals and Operators 52

Chapter 3. Unbounded Linear Operators 60
1. General Notions. Graph of an Operator 60
2. Closed Operators. Closable Operators 64
3. Adjoint Operator 68
4. Domination of Operators 72
5. Invariant Subspaces 75
6. Reducing Subspaces 78
7. Defect Number, Spectrum, and Resolvent of a Closed Operator 81
8. Skew Decompositions. Skew Reducibility 86

9. Spectral Theory of Compact Operators 89
10. Connection between the Spectral Properties of TS and ST 93

Chapter 4. Symmetric and Isometric Operators 97
1. Symmetric and Self-adjoint Operators. Deficiency Indices 97
2. Isometric and Unitary Operators 100
3. Cayley Transform 103
4. Extensions of Symmetric Operators. Von Neumann's Formulae 105
5. The Operator T^*T. Normal Operators 108
6. Classification of Spectral Points 110
7. Multiplication by the Independent Variable 112
8. Differentiation Operator 116

Chapter 5. Spectral Measure. Integration 123
1. Basic Notions 123
2. Extension of a Spectral Measure. Product Measures 126
3. Integral with Respect to a Spectral Measure. Bounded Functions 130
4. Integral with Respect to a Spectral Measure. Unbounded Functions 133
5. An Example of Commuting Spectral Measures whose Product is not Countably Additive 138

Chapter 6. Spectral Resolutions 140
1. Statements of Spectral Theorems. Functions of Operators 140
2. Spectral Theorem for Unitary Operators 145
3. Spectral Theorem for Self-adjoint Operators 148
4. Spectral Resolution of a One-parameter Unitary Group 150
5. Joint Spectral Resolution for a Finite Family of Commuting Self-adjoint Operators 153
6. Spectral Resolutions of Normal Operators 156

Chapter 7. Functional Model and the Unitary Invariants of Self-adjoint Operators 159
1. Direct Integral of Hilbert Spaces 159
2. Multiplication Operators and Decomposable Operators 164
3. Generating Systems and Spectral Types 169
4. Unitary Invariants of Spectral Measure 173
5. Unitary Invariants of Self-adjoint Operators 176
6. Decomposition of a Spectral Measure into the Absolutely Continuous and the Singular Part 179

Chapter 8. Some Applications of Spectral Theory 183
1. Polar Decomposition of a Closed Operator 183
2. Differential Equations of Evolution on Hilbert Space 187

3. Fourier Transform 192
4. Multiplications on L_2 ($\mathbf{R}^m$, $\mathbf{C}^m$) 194
5. Differential Operators with Constant Coefficients 197
6. Examples of Differential Operators 201

Chapter 9. Perturbation Theory 206
1. Essential Spectrum. Compact Perturbations 206
2. Compact Self-adjoint and Normal Operators 209
3. Finite-dimensional Perturbations and Extensions 214
4. Continuous Perturbations 217

Chapter 10. Semibounded Operators and Forms 221
1. Closed Positive Definite Forms 221
2. Semibounded Forms 224
3. Friedrichs Method of Extension of a Semibounded Operator to a Self-adjoint Operator 228
4. Fractional Powers of Operators. The Heinz Inequality 231
5. Examples of Quadratic Forms. The Sturm–Liouville Operator on [–1, 1] 233
6. Examples of Quadratic Forms. One-dimensional Schrödinger Operator 238

Chapter 11. Classes of Compact Operators 242
1. Canonical Representation and Singular Numbers of Compact Operators 242
2. Nuclear Operators. Trace of an Operator 245
3. Hilbert-Schmidt Operators 250
4. $\mathbf{S}_p$ Classes 253
5. Additional Information on Singular Numbers of Compact Operators 257
6. Σ_p Classes 262
7. Lidskii's Theorem 268
8. Examples of Compact Operators 271

Chapter 12. Commutation Relations of Quantum Mechanics 279
1. Statement of the Problem. Auxiliary Material 279
2. Properties of (B)-systems and (C)-systems 283
3. Representations of the Bose Relations. The Case $m = 1$ 287
4. Representations of the Bose Relations. General Case 291
5. Representations of the Canonical Relations 294

References 297

Subject Index 299