

Contents

	Acknowledgments	xi
	Introduction	3
CHAPTER I.	Distributions and Sobolev Spaces	10
§1.	Distributions	10
§2.	The Fourier Transform	14
§3.	Sobolev Spaces on \mathbf{R}^n	18
§4.	The Complex Interpolation Method	22
§5.	Sobolev Spaces on Bounded Domains and Compact Manifolds	24
§6.	Sobolev Spaces, L^p Style	30
§7.	Local Solvability of Constant Coefficient <i>PDE</i>	33
CHAPTER II.	Pseudodifferential Operators	36
§1.	The Fourier Integral Representation and Symbol Classes	36
§2.	The Pseudolocal Property	39
§3.	Asymptotic Expansions of a Symbol	40
§4.	Adjoint and Products	45
§5.	Coordinate Changes: Operators on a Manifold	47
§6.	L^2 and Sobolev Space Continuity	49
§7.	Families of Pseudodifferential Operators: Friedrichs' Mollifiers	53
§8.	Gårding's Inequality	54
§9.	References to Further Work	55
CHAPTER III.	Elliptic and Hypoelliptic Operators	60
§1.	Elliptic Operators	60
§2.	Hypoelliptic Operators with Constant Strength	62
§3.	Hypoelliptic Operators with Slowly Varying Strength	68
CHAPTER IV.	The Initial Value Problem and Hyperbolic Operators	70
§1.	Reduction to a First Order System	70
§2.	Symmetric Hyperbolic Systems	73

§3.	Strictly Hyperbolic Equations	76
§4.	Finite Propagation Speed: Finite Domain of Dependence	79
§5.	Quasilinear Hyperbolic Systems	82
§6.	The Vibrating Membrane Problem	92
§7.	Parabolic Evolution Equations	95
§8.	References to Further Work	100
CHAPTER V.	Elliptic Boundary Value Problems	101
§1.	Reduction to First Order Systems and Decoupling	102
§2.	A Priori Estimates and Regularity Theorems	104
§3.	Closed Range and Fredholm Properties	108
§4.	Regular Boundary Value Problems	114
§5.	Reduction of a Boundary Value Problem to a Regular One	119
CHAPTER VI.	Wave Front Sets and Propagation of Singularities	126
§1.	The Wave Front Set of a Distribution	126
§2.	Propagation of Singularities: The Hamilton Flow	130
§3.	Local Solvability	133
§4.	Systems: An Exponential Decay Result	135
CHAPTER VII.	The Sharp Gårding Inequality	138
§1.	A Multiple Symbol	138
§2.	Friedrichs' Symmetrization: Proof of the Sharp Gårding Inequality	141
CHAPTER VIII.	Geometrical Optics and Fourier Integral Operators	146
§1.	Egorov's Theorem	147
§2.	Propagation of Singularities	152
§3.	The Geometrical Optics Construction	155
§4.	Parametrix for Elliptic Evolution Equations	162
§5.	Fourier Integral Operators	166
§6.	Operators with Singular Phase Functions	181
§7.	The Fundamental Asymptotic Expansion Lemma	184
§8.	Egorov's Theorem for $OPS_{1/2,1/2}^m$	186
CHAPTER IX.	Reflection of Singularities	192
§1.	Decoupling First Order Systems	195
§2.	Elliptic Evolution Equations	197
§3.	Reflection of Singularities	201

CHAPTER X.	Grazing Rays and Diffraction	215
§1.	The Ansatz	216
§2.	Fourier-Airy Integral Operators	222
§3.	The Eikonal and Transport Equations	228
§4.	Justification and Analysis of the Parametrix	236
§5.	The Neumann Operator	242
§6.	The Kirchhoff Approximation	249
§7.	References to Further Work	257
CHAPTER XI.	L^p and Hölder Space Theory of Pseudodifferential Operators	261
§1.	Fourier Multipliers on L^p and Hölder Spaces	261
§2.	L^p and C^α Behavior of Operators in $OPS_{1,0}^m$	265
§3.	L^p Behavior of $OPS_{1,\delta}^0$	271
§4.	The Algebras $OP\mathcal{M}_\rho^m$ and $OP\mathcal{N}_\rho^m$ on L^p	275
§5.	Besov Spaces and Boundary Regularity	286
§6.	References to Further Work	291
CHAPTER XII.	Spectral Theory and Harmonic Analysis of Elliptic Self-Adjoint Operators	293
§1.	Functions of Elliptic Self-Adjoint Operators	295
§2.	The Asymptotic Behavior of the Spectrum	302
§3.	Poisson-like Kernels	308
§4.	Convergence of Eigenfunction Expansions	317
§5.	Eigenfunction Expansions of Measures	323
§6.	Harmonic Analysis on Compact Lie Groups	329
	A. Multipliers on G	330
	B. Asymptotic Behavior of Multiplicities	332
§7.	Some Tauberian Theorems	335
CHAPTER XIII.	The Calderon-Vaillancourt Theorem and Hörmander-Melin Inequalities	343
§1.	L^2 Continuity of $OPS_{0,0}^0(\mathbf{R}^n)$	346
§2.	L^2 Boundedness of $OPS_{\rho,\rho}^0(\mathbf{R}^n)$, $0 < \rho < 1$	348
§3.	L^2 Continuity of Other Sets of Operators	351
§4.	Hörmander-Melin Inequalities	352
CHAPTER XIV.	Uniqueness in the Cauchy Problem	358
§1.	Carleman Estimates	359
§2.	Reduction to Subelliptic Estimates, and Proof of UCP	360
§3.	UCP , Global Solvability, and All That	370

CHAPTER XV.	Operators with Double Characteristics	372
§1.	Hypoelliptic Operators	372
§2.	The Subprincipal Symbol and Microlocal Equivalence of Operators	389
§3.	Characteristics with Involution Self-intersection	399
§4.	Characteristics with Noninvolution Self-intersection	405
§5.	Characteristics with Conical Singularities and Conical Refraction	414
	A. The Equations of Crystal Optics	414
	B. Conical Refraction at a Flat Boundary	420
	Bibliography	424
	General Index	447
	Index of Symbols	451