

Table of Contents

Introduction	1
---------------------------	---

Chapter 1 The Palm-Martingale Calculus of Point Processes

Introduction	5
1. Stationary Marked Point Processes	6
1.1 The Canonical Space of a Point Process on the Real Line ..	6
1.2 Stationary Point Processes	8
1.3 Marks of a Point Process	10
1.4 Two Properties of Stationary Point Processes	14
2. Intensity	15
2.1 The Intensity of a Stationary Point Process	15
2.2 Campbell's Measure	15
3. Palm Probability	16
3.1 Matthes' Definition in Terms of Counting	16
3.2 Invariance of the Palm Probability	18
3.3 Mecke's Formula	19
3.4 Neveu's Exchange Formula	21
3.5 Miyazawa's Conservation Principle	23
4. From Palm Probability to Stationary Probability	23
4.1 The Inversion Formula of Ryll-Nardzewski and Slivnyak ..	23
4.2 Forward and Backward Recurrence Times	24
4.3 The Mean Value Formulas	26
4.4 The Inverse Construction of Slivnyak	27
5. Examples	32
5.1 Superposition of Independent Point Processes	32
5.2 Selected Marks and Conditioning	33
5.3 Selected Transitions of a Stationary Markov Chain	35

5.4	Delayed Marked Point Process	37
6.	Local Aspects of Palm Probability	38
6.1	The Korolyuk-Dobrushin Infinitesimal Estimates	38
6.2	Conditioning at a Point: Ryll-Nardzewski's Interpretation	39
7.	Ergodicity of Point Processes	40
7.1	Invariant Events	40
7.2	Ergodicity under the Stationary Probability and under the Palm Probability	42
7.3	The Cross Ergodic Theorems	42
7.4	Palm Theory in Discrete Time	43
8.	Stochastic Intensity	45
8.1	Predictable Processes	45
8.2	Stochastic Intensity Kernels	47
8.3	Stochastic Intensity Integration Formula and Martingales	49
8.4	Watanabe's Characterization of Poisson Processes	50
9.	The Connection Between Palm Probability and Stochastic Intensity	53
9.1	Invariance of Stochastic Intensity	53
9.2	Papangelou's Formula	54
9.3	Further Connection with Martingale Theory	57
10.	Poisson Imbedding	59
10.1	Macrostate Models of Markov Chains	59
10.2	Imbedded Thinning	62
	Bibliographical Comments	64

Chapter 2 Stationarity and Coupling

	Introduction	67
1.	Stability of the $G/G/1/\infty$ Queue	68
1.1	The $G/G/1/\infty$ Queue	68
1.2	Loynes' Stability Theorem	70
1.3	Construction Points and Cycles	71
2.	Proof of Loynes' Theorem	74
2.1	Reduction to the Palm Setting	74
2.2	Construction of the Workload Sequence	75

2.3	Uniqueness of the Stationary Workload	77
2.4	Construction Points	79
2.5	A Queueing Proof of the Ergodic Theorem	79
3.	The $G/G/s/\infty$ Queue	81
3.1	The Ordered Workload Vector	81
3.2	Existence of a Finite Stationary Workload	82
3.3	The Maximal Solution	83
4.	Coupling	87
4.1	Coupling and Convergence in Variation	87
4.2	Coupling of Stochastic Recurrent Sequences	89
4.3	Strong Coupling and Borovkov's Theory of Renovating Events	97
5.	Stability of the $G/G/1/0$ Queue	104
5.1	Counterexamples	104
5.2	Coupling in the $G/G/1/0$ Queue	106
5.3	Construction of an Enriched Probability Space	108
6.	Other Queueing Systems	112
6.1	The $G/G/\infty$ Pure Delay System	112
6.2	Service Disciplines in $G/G/1/\infty$: the Vector of Residual Services	113
6.3	$G/G/1/\infty$ Queues with Vacations	116
6.4	$G/G/1/\infty$ Queues with Mutual Service	118
7.	Stability of Queueing Networks via Coupling	119
7.1	$G/G/1/\infty$ Queues in Tandem	119
7.2	Kelly Type Networks	120
8.	Stability of Queueing Networks via Recurrence Equations	123
8.1	Finite Capacity Queues in Tandem with Blocking	123
8.2	Existence of a Stationary Solution	125
8.3	Uniqueness of the Stationary Solutions	128
9.	The Saturation Rule	130
9.1	The Monotone Separable Framework	130
9.2	Proof of the Saturation Rule	132
9.3	Examples	135
	Bibliographical Comments	137

Chapter 3 Formulas

Introduction	141
1. Little's Formulas	142
1.1 The θ_t -Framework for Stationary Queueing Systems	142
1.2 $L = \lambda W$	145
1.3 The Swiss Army Formula	150
2. $H = \lambda G$	156
2.1 The Function Space Campbell-Little-Mecke Formula and the $H = \lambda G$ Formula	156
2.2 Pollaczek-Khinchin's Mean Value Formulas	160
2.3 Kleinrock's Conservation Law	162
3. Event and Time Averages	165
3.1 PASTA	165
3.2 Applications of Papangelou's Formula	167
4. Formulas Derived from Conservation Equations	174
4.1 First Order Equivalence	174
4.2 Brill and Posner's Formula	179
4.3 Takać's Formula and Queues with Vacations	182
4.4 Backward and Forward Recurrence Time	186
5. Queueing Applications of the Stochastic Intensity Integration Formula	187
5.1 Reminder	187
5.2 Priorities in $M/GI/1/\infty$	188
Bibliographical Comments	194

Chapter 4 Stochastic Ordering and Comparison of Queues

Introduction	197
1. Comparison of Service Disciplines	199
1.1 Partial Orderings on \mathbb{R}^n	199
1.2 Optimality of the SRPT Discipline in $G/G/1/\infty$	201
1.3 Optimality of the FIFO Discipline	203
2. Comparison of Queues	209

2.1	Integral Stochastic Orderings	209
2.2	Analytical Characterizations	212
2.3	Strassen's Pointwise Representation Theorems	214
2.4	Comparison of Stochastic Recurrences	216
2.5	Bounds Based on Integral Orderings	217
2.6	Stability of Stochastic Orders by Limits	218
2.7	Comparison of Basic Queues	219
2.8	Other Queueing Systems	222
3.	Association Properties of Queues	224
3.1	Association of Random Variables	224
3.2	Bounds by Association	226
4.	Stochastic Comparison of Time-Stationary Queues	229
4.1	Comparison of Point Processes	229
4.2	Stochastic Comparison under Time-Stationary Probabilities	232
4.3	Comparison of Queues	236
5.	Proof of Strassen's Theorem	239
	Bibliographical Comments	242
	References	245
	Index	253