

Contents

Introduction	1
Part I. Adaptive Algorithms: Applications	7
1. General Adaptive Algorithm Form	9
1.1 Introduction	9
1.2 Two Basic Examples and Their Variants	10
1.3 General Adaptive Algorithm Form and Main Assumptions	23
1.4 Problems Arising	29
1.5 Summary of the Adaptive Algorithm Form: Assumptions (A)	31
1.6 Conclusion	33
1.7 Exercises	34
1.8 Comments on the Literature	38
2. Convergence: the ODE Method	40
2.1 Introduction	40
2.2 Mathematical Tools: Informal Introduction	41
2.3 Guide to the Analysis of Adaptive Algorithms	48
2.4 Guide to Adaptive Algorithm Design	55
2.5 The Transient Regime	75
2.6 Conclusion	76
2.7 Exercises	76
2.8 Comments on the Literature	100
3. Rate of Convergence	103
3.1 Mathematical Tools: Informal Description	103
3.2 Applications to the Design of Adaptive Algorithms with Decreasing Gain	110
3.3 Conclusions from Section 3.2	116
3.4 Exercises	116
3.5 Comments on the Literature	118

4. Tracking Non-Stationary Parameters	120
4.1 Tracking Ability of Algorithms with Constant Gain	120
4.2 Multistep Algorithms	142
4.3 Conclusions	158
4.4 Exercises	158
4.5 Comments on the Literature	163
5. Sequential Detection; Model Validation	165
5.1 Introduction and Description of the Problem	166
5.2 Two Elementary Problems and their Solution	171
5.3 Central Limit Theorem and the Asymptotic Local Viewpoint	176
5.4 Local Methods of Change Detection	180
5.5 Model Validation by Local Methods	185
5.6 Conclusion	188
5.7 Annex: Proofs of Theorems 1 and 2	188
5.8 Exercises	191
5.9 Comments on the Literature	197
6. Appendices to Part I	199
6.1 Rudiments of Systems Theory	199
6.2 Second Order Stationary Processes	205
6.3 Kalman Filters	208
Part II. Stochastic Approximations: Theory	211
1. O.D.E. and Convergence A.S. for an Algorithm with Locally Bounded Moments	213
1.1 Introduction of the General Algorithm	213
1.2 Assumptions Peculiar to Chapter 1	219
1.3 Decomposition of the General Algorithm	220
1.4 L^2 Estimates	223
1.5 Approximation of the Algorithm by the Solution of the O.D.E. ..	230
1.6 Asymptotic Analysis of the Algorithm	233
1.7 An Extension of the Previous Results	236
1.8 Alternative Formulation of the Convergence Theorem	238
1.9 A Global Convergence Theorem	239
1.10 Rate of L^2 Convergence of Some Algorithms	243
1.11 Comments on the Literature	249

2. Application to the Examples of Part I	251
2.1 Geometric Ergodicity of Certain Markov Chains	251
2.2 Markov Chains Dependent on a Parameter θ	259
2.3 Linear Dynamical Processes	265
2.4 Examples	270
2.5 Decision-Feedback Algorithms with Quantisation	276
2.6 Comments on the Literature	288
3. Analysis of the Algorithm in the General Case	289
3.1 New Assumptions and Control of the Moments	289
3.2 L^q Estimates	293
3.3 Convergence towards the Mean Trajectory	298
3.4 Asymptotic Analysis of the Algorithm	301
3.5 "Tube of Confidence" for an Infinite Horizon	305
3.6 Final Remark. Connections with the Results of Chapter 1	306
3.7 Comments on the Literature	306
4. Gaussian Approximations to the Algorithms	307
4.1 Process Distributions and their Weak Convergence	308
4.2 Diffusions. Gaussian Diffusions	312
4.3 The Process $U^\gamma(t)$ for an Algorithm with Constant Step Size	314
4.4 Gaussian Approximation of the Processes $U^\gamma(t)$	321
4.5 Gaussian Approximation for Algorithms with Decreasing Step Size	327
4.6 Gaussian Approximation and Asymptotic Behaviour of Algorithms with Constant Steps	334
4.7 Remark on Weak Convergence Techniques	341
4.8 Comments on the Literature	341
5. Appendix to Part II: A Simple Theorem in the "Robbins-Monro" Case	343
5.1 The Algorithm, the Assumptions and the Theorem	343
5.2 Proof of the Theorem	344
5.3 Variants	345
Bibliography	349
Subject Index to Part I	361
Subject Index to Part II	364