

Contents

Preface to the first edition	IX
Preface to the English edition	XI

Chapter 1

FUNDAMENTALS OF FUNCTIONAL ANALYSIS

§ 1	<i>Convexity in topological linear spaces</i>	1
1.1	Classes of topological linear spaces	1
1.2	Convex sets	7
1.3	Separation of convex sets	14
§ 2	<i>Duality in linear normed spaces</i>	23
2.1	Dual systems of the linear spaces	23
2.2	Weak topologies on linear normed spaces	27
2.3	Reflexive Banach spaces	32
2.4	Duality mapping	35
§ 3	<i>Vector-valued functions and distributions</i>	39
3.1	The Bochner integral	40
3.2	Bounded variation vector functions	41
3.3	Vector distributions on the real axis	44
3.4	Vector distributions and $W^{k,p}$ spaces	49
3.5	Sobolev spaces	53
§ 4	<i>Maximal monotone operators</i>	56
4.1	Definitions and fundamental results	56
4.2	Evolution equations in Hilbert spaces	63

Chapter 2

CONVEX FUNCTIONS

§ 1	<i>General properties of convex functions</i>	73
1.1	Definitions and basic properties	73
1.2	Lower-semicontinuous functions	75
1.3	Lower-semicontinuous convex functions	77
1.4	Conjugate functions	82
§ 2	<i>The subdifferential of a convex function</i>	89
2.1	Definition and fundamental results	89
2.2	Further properties of subdifferential mappings	95
2.3	Regularization of the convex function	104
2.4	Perturbations of cyclically monotone operators	109
2.5	Variational inequalities	118
§ 3	<i>Concave-convex functions</i>	128
3.1	Saddle points and minimax equality	128
3.2	Saddle functions	130
3.3	Minimax theorems	140
	Bibliographical notes	150

Chapter 3

CONVEX PROGRAMMING

§ 1	<i>Optimality conditions</i>	152
1.1	The case of a finite number of constraints	152
1.2	Operatorial convex constraints	159

1.3	Non-linear programming in the case of Fréchet-differentiability	163
1.4	Examples	173
§ 2	<i>Duality in convex programming</i>	178
2.1	Dual problems	178
2.2	Fenchel duality theorem	186
2.3	Examples	191
§ 3	<i>Applications of the duality theory</i>	201
3.1	Linear programming	201
3.2	The best approximation problem	205
	Bibliographical notes	211

Chapter 4

CONVEX CONTROL PROBLEMS IN HILBERT SPACES

§ 1	<i>Necessary and sufficient conditions for optimality</i>	214
1.1	Basic assumptions	214
1.2	Optimality theorem	218
1.3	Proof of Theorem 1.1	222
1.4	Proof of Theorem 1.2	239
1.5	Further remarks on optimality theorems	243
§ 2	<i>The dual optimal control problem</i>	245
2.1	Formulation of the dual problem	245
2.2	The duality theorem	248
2.3	Some examples	250

§ 3	<i>Convex control problems associated with linear evolutionary processes in Hilbert space</i>	265
3.1	Statement of the problem	266
3.2	The optimality theorem	269
3.3	Optimal control of linear hereditary systems	275
§ 4	<i>Synthesis of optimal control</i>	286
4.1	Optimal synthesis function	286
4.2	Hamilton-Jacobi equations	293
	Bibliographical notes	300
	<i>Bibliography</i>	302
	<i>Subject index</i>	313