

# Contents

<b>Chapter 1</b>	<b>Topics in Functional Analysis</b>	<b>1</b>
1.0	Introduction	1
1.1	Set Theory	2
1.2	Functions	5
1.3	Matrices	7
1.4	Solving Matrix Systems	9
1.5	Metric Spaces	18
1.6	Linear Spaces	22
1.7	Normed Linear Spaces	25
1.8	Approximations	31
<b>Chapter 2</b>	<b>Integration Theory</b>	<b>37</b>
2.0	Introduction	37
2.1	Reimann and Lebesgue Integrals: Step and Simple Functions	37
2.2	Lebesgue Measure	38
2.3	Measurable Functions	40
2.4	The Lebesgue Integral	41
	2.4.1 Bounded Functions	42
	2.4.2 Unbounded Functions	44
2.5	Key Theorems in Integration Theory	47
2.6	$L_p$ Spaces	49
	2.6.1 $m$ -Equivalent Functions	49
	2.6.2 The Space $L_p$	50
2.7	The Metric Space, $L_p$	51
2.8	Convergence of Sequences	51
	2.8.1 Common Modes of Convergence	51
	2.8.2 Convergence in $L_p$	52
	2.8.3 Convergence in Measure (M)	52
	2.8.4 Almost Uniform Convergence (AU)	52
	2.8.5 Is the Approximation Converging?	52
	2.8.6 Counterexamples	53
2.9	Capsulation	55
<b>Chapter 3</b>	<b>Hilbert Space and Generalized Fourier Series</b>	<b>57</b>
3.0	Introduction	57
3.1	Inner Product and Hilbert Space	58
3.2	Best Approximations in an Inner Product Space	62
3.3	Approximations in $L_2(E)$	70
	3.3.1 Parseval's Identity	71
	3.3.2 Bessel's Inequality	71

3.4	Vector Representations and Best Approximations	71
3.5	Computer Program	82
<b>Chapter 4</b>	<b>Linear Operators</b>	<b>89</b>
4.0	Introduction	89
4.1	Linear Operator Theory	89
4.2	Operator Norms	93
4.3	Examples of Linear Operators in Engineering	97
4.4	Superposition	101
<b>Chapter 5</b>	<b>The Best Approximation Method</b>	<b>104</b>
5.0	Introduction	104
5.1	An Inner Product for the Solution of Linear Operator Equations	104
5.2	Definition of Inner Product and Norm	106
5.3	Generalized Fourier Series	108
5.4	Approximation Error Evaluation	117
5.5	The Weighted Inner Product	124
5.6	Considerations in Choosing Basis Functions	128
5.6.1	Global Basis Elements	128
5.6.2	Spline Basis Functions	129
5.6.3	Mixed Basis Functions	133
<b>Chapter 6</b>	<b>The Best Approximation Method: Applications</b>	<b>134</b>
6.0	Introduction	134
6.1	Sensitivity of Computational Results to Variation in the Inner Product Weighting Factor	134
6.2	Solving Two-Dimensional Potential Problems	137
6.3	Application to Other Linear Operators	146
6.4	Computer Program: Two-Dimensional Potential Problems Using Real Variable Basis Functions	150
6.4.1	Introduction	150
6.4.2	Input Data Description	152
6.4.3	Computer Program Listing	154
6.5	Application of Computer Program	166
6.5.1	A Fourth Order Differential Equation	167
<b>Chapter 7</b>	<b>Solving Potential Problems using the Best Approximation Method</b>	<b>170</b>
7.0	Introduction	170
7.1	The Complex Variable Boundary Element Method	171
7.1.1	Objectives	171
7.1.2	Definition 7.1.1 (Working Space, $W_0$ )	171
7.1.3	Definition 7.1.2 (the Function $  \omega  $ to $  \omega  _2$ )	172

7.1.4	Almost Everywhere (ae) Equality	172
7.1.5	Theorem (relationship of $  \omega  $ to $  \omega  _2$ )	172
7.1.6	Theorem	173
7.1.7	Theorem	173
7.2	Mathematical Development	174
7.2.1	Discussion: (A Note on Hardy Spaces)	174
7.2.2	Theorem (Boundary Integral Representation)	174
7.2.3	Almost Everywhere (ae) Equivalence	174
7.2.4	Theorem (Uniqueness of Zero Element in $W_0$ )	175
7.2.5	Theorem ( $W_0$ is a Vector Space)	175
7.2.6	Theorem (Definition of the Inner-Product)	176
7.2.7	Theorem ( $W_0$ is an Inner-Product Space)	176
7.2.8	Theorem ( $  \omega  $ is a Norm on $W_0$ )	176
7.2.9	Theorem	176
7.3	The CVBEM and $W_0$	176
7.3.1	Definition 7.3.1 (Angle Points)	176
7.3.2	Definition 7.3.2 (Boundary Element)	177
7.3.3	Theorem	177
7.3.4	Definition 7.3.3 (Linear Basis Function)	177
7.3.5	Theorem	177
7.3.6	Definition 7.3.4 (Global Trial Function)	177
7.3.7	Theorem	178
7.3.8	Discussion	178
7.3.9	Theorem	178
7.3.10	Discussion	178
7.3.11	Theorem (Linear Independence of Nodal Expansion Functions)	180
7.3.12	Discussion	181
7.3.13	Theorem	181
7.3.14	Theorem	182
7.3.15	Discussion	182
7.4	The Space $W_0^\wedge$	183
7.4.1	Definition 7.4.1 ( $W_0^\wedge$ )	183
7.4.2	Theorem	183
7.4.3	Theorem	183
7.4.4	Discussion	184
7.4.5	Theorem	184
7.4.6	Theorem	185
7.4.7	Discussion: Another Look at $W_0$	185
7.5	Applications	185
7.5.1	Introduction	185
7.5.2	Nodal Point Placement on $\Gamma$	186
7.5.3	Potential Flow-Field (Flow-Net) Development	186
7.5.4	Approximate Boundary Development	186
7.5.5	Application Problems	187

7.6	Computer Program: Two-Dimensional Potential Problems using Analytic Basis Functions (CVBEM)	187
7.6.1	Introduction	187
7.6.2	CVBEM1 Program Listing	191
7.6.3	Input Variable Description for CVBEM1	203
7.6.4	CVBEM2 Program Listing	204
7.7	Modelling Groundwater Contaminant Transport	213
7.7.1	Application 1A	214
7.7.2	Application 1B	214
7.7.3	Application 2A	214
7.7.4	Application 2B	214
7.8	Three Dimensional Potential Problems	217
7.8.1	Approximation Error Evaluation - Approximate Boundary Method	217
7.8.2	Computer Implementation	218
7.8.3	Application	219
7.8.4	Trial Functions	219
7.8.5	Constructing the Approximate Boundary, $\hat{\Gamma}$	221
<b>Chapter 8</b>	<b>Applications to Linear Operator Equations</b>	<b>222</b>
8.0	Introduction	222
8.1	Data Fit Analysis	222
8.2	Ordinary Differential Equations	223
8.3	Best Approximation of Function	226
8.4	Matrix Systems	228
8.5	Linear Partial Differential Equations	230
8.6	Linear Integral Equations	233
8.6.1	An Inverse Problem	234
8.6.2	Best Approximation of the Transfer Function in a Linear Space	236
<b>References</b>		<b>238</b>
<b>Appendix A</b>	<b>Derivation of CVBEM Approximation Function</b>	<b>239</b>
<b>Appendix B</b>	<b>Convergence of CVBEM Approximator</b>	<b>243</b>
<b>Appendix C</b>	<b>The Approximate Boundary for Error Analysis</b>	<b>245</b>
<b>Index</b>		<b>249</b>