

Contents

	page
Preface	ix
Contents	v

Chapter I Elements of Spectral Theory with Examples

1. Some Function Spaces and their Properties	1
1.1. Preliminary Notations	2
1.2. Trace Results and Green's Formulae	3
1.3. A Compactness Criterion	4
1.4. Poincaré Type Inequalities	5
1.5. Properties of the Spaces $H^m(\Omega)$	6
1.6. The Space $H(\Omega; \Delta)$	7
1.7. Lax-Milgram Lemma	8
1.8. Hilbert Space-Valued Distributions	9
1.9. Analytic Functions with Values in a Banach Space	12
2. Some Classical Examples of Vibrating Systems	14
2.1. Vibrations of a Drum	15
2.2. Spectral Problems with Neumann and Robin Boundary Conditions	18
2.3. Steklov Eigenvalue Problem	20
2.4. The Acoustic Resonator	21
2.5. Propagation of Waves in Free Space	23
2.6. Vibrations of Air Trapped in many Narrow Channels	26
3. Spectral Theory of Linear Operators	30
3.1. Generalities on Unbounded Linear Operators	31
3.2. Self-Adjoint Operators	34
3.3. Compact Operators	39
3.4. Compact Self-Adjoint Operators	44
3.5. Spectral Decomposition of Self-Adjoint Operators	49
3.6. Elementary Properties of Spectral Families	65

4. Effects of Perturbations	69
4.1. Continuity Properties of Eigenvalues	70
4.2. Perturbations by Operators of Finite Rank	73
4.3. Quadratic Eigenvalue Problems	76
4.4. Convergence of Spectral Families	83

Chapter II

Spectral Problems in Fluid-Solid Structures

1. Mathematical Models of Vibrations of Fluid-Solid Structures	92
1.1. The Case of a Perfect Incompressible Fluid	94
1.2. The Case of a Slightly Compressible Perfect Fluid	96
1.3. The Case of an Incompressible Viscous Fluid	98
1.4. The Case of a Slightly Compressible Viscous Fluid	99
1.5. Other Cases	100
2. Existence Results	100
2.1. The Laplace Model	101
2.2. The Helmholtz Model	106
2.3. The Stokes Model	113
3. Bounds on Eigenvalues	123
3.1. Bounds for the Laplace Model	124
3.2. Interlacing Inequalities for the Helmholtz Model	129
3.3. More on the Eigenvalues of the Helmholtz Model	132
3.3.1. A New Eigenvalue Problem	133
3.3.2. A Homotopy between Laplace and Helmholtz Models .	136
3.3.3. Continuity Properties of the Homotopy	137
3.3.4. A Geometric Condition and a Consequence	143
3.4. Bounds for the Stokes Model	144
4. Numerical Methods in Fluid-Solid Structures	150
4.1. Technique of Added Mass Matrix for the Laplace Model . . .	151
4.2. A Numerical Example Concerning Laplace Model	154
4.3. Unitary Operator Method	154
4.4. Practical Use of the Unitary Operator Method	165
4.5. Application to Helmholtz Model	167
4.6. A Numerical Example Concerning Stokes Model	171

Chapter III

Asymptotic Methods in Fluid-Solid Structures

1.	Beppo-Levi Spaces and their Properties	174
	1.1. The Space $BL(\Omega)$	175
	1.2. The Space $W^1(\Omega)$	177
	1.3. Behaviour at Infinity	181
2.	Bloch Wave Method in a Classical Example	184
	2.1. Bloch Waves and their Properties	185
	2.2. Momentum Space Picture of Bloch Wave Decomposition . .	196
	2.3. Absence of Eigenvalues for Schrödinger Operator	198
	2.4. Non-Standard Homogenization of Spectral Problems	204
3.	Bloch Wave Method in the Laplace Model	209
	3.1. Discrete Bloch Wave Decomposition of the Laplace Model .	210
	3.1.1. Description of the Spectrum	211
	3.1.2. The Orthogonal Decomposition	213
	3.1.3. Splitting into Subproblems in Y^*	217
	3.2. Continuous Bloch Wave Decomposition of the Laplace Model	219
	3.3. Consequences of the Bloch Wave Decomposition of S . .	237
	3.4. Convergence Analysis	240
	3.5. Numerical Results in Bloch Wave Decomposition	246
	3.6. Analyticity Properties of the Eigenvalues	248
	3.7. Behaviour of the Eigenvalues at the Origin	253
4.	Bloch Wave Method in the Helmholtz Model	260
	4.1. Description of the Limit Operator	260
	4.2. Bloch Wave Decomposition of the Limit Operator	269
	4.3. Convergence Analysis	273
	4.3.1. Main Convergence Result	273
	4.3.2. The Wave Equation associated with B_n	275
	4.3.3. Passage to the Limit	277
	4.3.4. Proof of the Main Result	279
5.	Two-Scale Convergence Method	280
	5.1. Basic Results	281
	5.2. Application to a Classical Example	289
	5.3. Homogenization of the Laplace Model	294
	5.4. Homogenization of the Helmholtz Model	306
6.	Asymptotic Expansions in Fluid-Solid Structures	312
	6.1. Two-Scale Expansion for the Laplace Model	312
	6.2. Two-Scale Expansion for the Helmholtz Model	316

Open Questions	319
References	321
Subject Index	329