CONTENTS

1 Presentation of the equations 1
1.1 What is a perfect fluid? 1
1.2 From Lagrange to Euler 4
1.3 Vorticity, pressure and dimension 2 8
1.4 References and remarks 15

2 Littlewood–Paley theory 16
2.1 Dyadic decomposition 16
2.2 Sobolev spaces 20
2.3 Hölder spaces 26
2.4 Paradifferential calculus 32
2.5 The pressure and its gradient field 37
2.6 References and remarks 43

3 Concerning Biot–Savart's law 44
3.1 L^p estimates 44
3.2 L^∞ estimates: some examples 47
3.3 Riesz operators and bounded functions 52
3.4 References and remarks 63

4 The case of smooth initial data 65
4.1 Resolution of a model problem 65
4.2 Application to Euler's equation 76
4.3 References and remarks 83

5 The case of bounded vorticity 85
5.1 Yudovich's theorem 85
5.2 On ordinary differential equations 90
5.3 An example 94
5.4 The vortex patch problem 96
5.5 Proof of the persistence 97
5.6 References and remarks 106

6 Vortex sheets 109
6.1 Presentation of the problem 109
6.2 The study of the function G 111
6.3 The limit 112
6.4 References and remarks 117

7 The wave front and the product 119

7.1 Presentation of the wave front 119
7.2 When can the product be defined? 125
7.3 Analytic and Gevrey wave-front sets 130
7.4 References and remarks 139

8 Analyticity and Gevrey regularity 140
8.1 Statement of the theorems 140
8.2 Multilinear operators 141
8.3 Regularity along the flow lines 150
8.4 References and remarks 156

9 Singular vortex patches 157
9.1 Presentation of the problem 157
9.2 Local Littlewood–Paley theory 161
9.3 Dynamics of singular patches 169
9.4 References and remarks 180

References 181

Index 186