

TABLE OF CONTENTS

CHAPTER 1	ORIENTATION	1
1.1	Discrete space, continuous time, random walk	1
1.2	Discrete space, discrete time	10
1.3	Circular space, continuous time	11
1.4	Continuous space, continuous time	15
1.5	Markov process	17
1.6	Dynkin's formula	23
CHAPTER 2	POPULATION GENETICS MODELS	24
2.1	Wright's model	24
2.2	Feller's model	25
2.3	Moran's model	26
2.4	Variable population size	26
2.5	Wright's model with mutation	28
2.6	A model of irreversible mutation or a model of infinite alleles	29
2.7	A selection model	29
2.8	A model of dominance	31
2.9	Birth-and-death process	31
2.10	Density or frequency dependent process . .	32
2.11	Time inhomogeneous process	33
2.12	A model of random environment	34
CHAPTER 3	CLASSIFICATION OF BOUNDARIES	36
3.1	Regular boundary	36
3.2	Exit boundary	37
3.3	Entrance boundary	37
3.4	Natural boundary	37
3.5	Nature of boundary	37
3.6	Examples	39
CHAPTER 4	EXPECTATION OF INTEGRATION ALONG SAMPLE PATHS	43
4.1	Integration along sample paths	43
4.2	The boundary conditions	46
4.3	An example	47
4.4	Green's function for a pure random process	49
4.5	Computer simulation	52
4.6	Sum of heterozygotes	54

4.7	Process with reflecting boundary	55
4.8	Irreversible mutation model (or infinite alleles)	59
4.9	General form of Green's function	62
4.10	Probability of fixation	63
4.11	Behavior of sample paths near the origin	65
4.12	Higher moments	67
4.13	Nagylaki's formula	70
CHAPTER 5	MODIFICATION OF PROCESSES	74
5.1	Killing and creating paths	74
5.2	Selection of paths	75
5.3	Random drift and fixation time	77
5.4	A case of genic selection	79
5.5	A symmetric property of sample paths	80
5.6	A general formula	81
5.7	Age of sample paths	82
5.8	Number of affected individuals and genetic load	87
5.9	Sojourn time of conditional sample paths on the present frequency	87
5.10	A conditional age	89
5.11	Computer simulation	91
5.12	Random time change	93
CHAPTER 6	NUMERICAL INTEGRATION OF THE KOLMOGOROV BACKWARD EQUATION	95
6.1	Integration method	95
6.2	Examples	98
CHAPTER 7	EIGENVALUES AND EIGENVECTORS OF THE KBE	102
7.1	Eigenvalues and eigenvectors	102
7.2	Pure random drift case	105
7.3	Mutation and random drift case	109
7.4	Irreversible mutation case	112
7.5	Hypergeometric differential equation	113
7.6	Orthogonality of eigenfunctions	114
7.7	Expansion by eigenfunctions	115
7.8	The steady-state distribution of gene frequencies	118

CHAPTER 8	APPROXIMATION METHODS	121
8.1	Perturbation	121
8.2	Examples	124
8.3	Numerical method	127
8.4	Singular perturbation	128
CHAPTER 9	GEOGRAPHICAL STRUCTURE OF POPULATIONS	130
9.1	One-dimensional populations, discrete colonies	130
9.2	Continuous space	137
9.3	Two-dimensional populations	140
9.4	Two-dimensional continuous space	142
9.5	Higher order moments	143
9.6	Numerical analysis at equilibrium	144
9.7	A differential equation and asymptotic formulae	147
9.8	Random drift	149
9.9	Time to fixation	154
CHAPTER 10	GEOGRAPHICALLY INVARIANT PROPERTIES	156
10.1	Discrete time model	156
10.2	Continuous time model	159
10.3	Markov process	165
10.4	Diffusion method	175
10.5	Computer simulation of gene frequency change	178
10.6	Invariance based on diffusion method	180
10.7	Computer simulation of heterozygote distribution and other invariant properties	184
CHAPTER 11	GENE FREQUENCY DISTRIBUTIONS AND RANDOM DRIFT IN GEOGRAPHICALLY STRUCTURED POPULATIONS	187
11.1	Gene frequency distribution (global) in a structured population	187
11.2	Distribution of local gene frequencies	195
11.3	Random drift in a structured population	197
CHAPTER 12	SOME SPECIAL PROBLEMS	201
12.1	Variance of homozygote probability for the infinite neutral allele model	201

12.2	Variance of homozygote probability in a geographically structured population	204
12.3	Number of alleles	206
12.4	Some properties of the stepwise muta- tion model	207
APPENDIX I	THE INFINITESIMAL MEAN CHANGE ($M_{\delta x}$) AND VARIANCE ($V_{\delta x}$)	218
A1.1	Mutation model I	218
A1.2	Mutation model II	219
A1.3	Derivation of Kolmogorov backward equa- tions (KBE)	221
APPENDIX II	A SUPPLEMENTARY NOTE ON THE EXISTENCE AND UNIQUENESS OF THE SOLUTION FOR THE RECUR- RENCE EQUATION (9.7)	229
APPENDIX III	DISTRIBUTION OF STOCHASTIC INTEGRALS . . .	234
REFERENCES	240