

Contents

1. Convex and Quadratic Programming	1
1.1 Introduction	1
1.1.1 The Linearization Algorithm	1
1.1.2 Convergence of the Algorithm	3
1.1.3 General Remarks	4
1.1.4 Notation	5
1.2 Necessary Conditions for a Minimum and Duality	6
1.2.1 Convex Sets	7
1.2.2 Convex Functions	7
1.2.3 Foundations of Convex Programming	10
1.2.4 Duality in Convex Programming	15
1.2.5 Necessary Conditions for Extrema. General Problem	16
1.2.6 Necessary Conditions for Extrema: Second Order Conditions	18
1.2.7 Minimax Problems	18
1.2.8 Penalty Functions	19
1.3 Quadratic Programming Problems	22
1.3.1 Conjugate Gradient Method	23
1.3.2 Conjugate Gradient Algorithm	25
1.3.3 Existence of a Solution	25
1.3.4 Necessary Conditions for an Extremum and the Dual Problem	28
1.3.5 Application, Projection onto a Subspace	30
1.3.6 Algorithm for the Quadratic Programming Problem	31
1.3.7 Computational Aspects	37
1.3.8 Algorithms for Simple Constraints. Generalization	41
2. The Linearization Method	43
2.1 The General Algorithm	43
2.1.1 Main Assumptions	44
2.1.2 Formulation of the Algorithm	44
2.1.3 Convergence of the Algorithm	45
2.1.4 Computational Aspects	49
2.1.5 Generalizations	51
2.1.6 The Linear Programming Problem	53
2.1.7 The Linearization Method with Equality-Type Constraints	57

2.1.8 Simple Constraints	58
2.1.9 Choice of Parameters in the Linearization Method. Modified Algorithm	60
2.2 Resolution of Systems of Equations and Inequalities	65
2.2.1 The Auxiliary Problem	66
2.2.2 The Algorithm	66
2.2.3 Convergence of the Algorithm	67
2.3 Acceleration of the Convergence of the Linearization Method	75
2.3.1 Main Assumptions	75
2.3.2 Local Analysis of the Auxiliary Problem	76
2.3.3 Preliminary Lemmas	81
2.3.4 The Linearization Algorithm and Acceleration of Convergence	84
2.3.5 Linear Transformations of the Problem	87
2.3.6 Modifications of the Linearization Method	90
3. The Discrete Minimax Problem and Algorithms	99
3.1 The Discrete Minimax Problem	99
3.1.1 The Auxiliary Problem	100
3.1.2 Some Bounds	102
3.1.3 Algorithms	104
3.1.4 Algorithm for $A_k = I_n$	106
3.1.5 Acceleration of Convergence in the Convex Case	111
3.2 The Dual Algorithm for Convex Programming Problems	114
3.2.1 The Dual Algorithm	115
3.2.2 Bounds on the Rate of Convergence.....	118
3.2.3 An Algorithm for Convex Programming Problems	123
3.3 Algorithms and Examples	129
3.3.1 The Linearization Method	129
3.3.2 The Accelerated Linearization Method	132
3.3.3 Examples of Calculations	134
Appendix: Comments on the Literature	143
References	145