

Contents

Preface	vii
Chapter 1 Metric and Normed Spaces	1
1.1 Metrics and norms	1
1.2 Convergence	6
1.3 Upper and lower bounds	9
1.4 Continuity	11
1.5 Open and closed sets	14
1.6 The completion of a metric space	17
1.7 Compactness	22
1.8 Maxima and minima	28
1.9 References	30
1.10 Exercises	30
Chapter 2 Continuous Functions	35
2.1 Convergence of functions	35
2.2 Spaces of continuous functions	37
2.3 Approximation by polynomials	40
2.4 Compact subsets of $C(K)$	44
2.5 Ordinary differential equations	51
2.6 References	58
2.7 Exercises	58
Chapter 3 The Contraction Mapping Theorem	61
3.1 Contractions	61
3.2 Fixed points of dynamical systems	63
3.3 Integral equations	67
3.4 Boundary value problems for differential equations	69
3.5 Initial value problems for differential equations	73
3.6 References	78

3.7	Exercises	78
Chapter 4 Topological Spaces		81
4.1	Topological spaces	81
4.2	Bases of open sets	84
4.3	Comparing topologies	86
4.4	References	88
4.5	Exercises	88
Chapter 5 Banach Spaces		91
5.1	Banach spaces	91
5.2	Bounded linear maps	95
5.3	The kernel and range of a linear map	102
5.4	Finite-dimensional Banach spaces	106
5.5	Convergence of bounded operators	108
5.6	Dual spaces	115
5.7	References	120
5.8	Exercises	120
Chapter 6 Hilbert Spaces		125
6.1	Inner products	125
6.2	Orthogonality	130
6.3	Orthonormal bases	133
6.4	Hilbert spaces in applications	141
6.5	References	143
6.6	Exercises	144
Chapter 7 Fourier Series		149
7.1	The Fourier basis	149
7.2	Fourier series of differentiable functions	157
7.3	The heat equation	161
7.4	Other partial differential equations	166
7.5	More applications of Fourier series	170
7.6	Wavelets	175
7.7	References	182
7.8	Exercises	182
Chapter 8 Bounded Linear Operators on a Hilbert Space		187
8.1	Orthogonal projections	187
8.2	The dual of a Hilbert space	190
8.3	The adjoint of an operator	193
8.4	Self-adjoint and unitary operators	197
8.5	The mean ergodic theorem	201

8.6	Weak convergence in a Hilbert space	204
8.7	References	211
8.8	Exercises	211

Chapter 9 The Spectrum of Bounded Linear Operators 215

9.1	Diagonalization of matrices	216
9.2	The spectrum	218
9.3	The spectral theorem for compact, self-adjoint operators	222
9.4	Compact operators	228
9.5	Functions of operators	232
9.6	Perturbation of eigenvalues	235
9.7	References	239
9.8	Exercises	239

Chapter 10 Linear Differential Operators and Green's Functions 245

10.1	Unbounded operators	245
10.2	The adjoint of a differential operator	249
10.3	Green's functions	253
10.4	Weak derivatives	259
10.5	The Sturm-Liouville eigenvalue problem	268
10.6	Laplace's equation	273
10.7	References	280
10.8	Exercises	281

Chapter 11 Distributions and the Fourier Transform 287

11.1	The Schwartz space	287
11.2	Tempered distributions	291
11.3	Operations on distributions	294
11.4	The convergence of distributions	298
11.5	The Fourier transform of test functions	301
11.6	The Fourier transform of tempered distributions	306
11.7	The Fourier transform on L^1	307
11.8	The Fourier transform on L^2	311
11.9	Translation invariant operators	313
11.10	Green's functions	316
11.11	The Poisson summation formula	319
11.12	The central limit theorem	320
11.13	References	328
11.14	Exercises	328

Chapter 12 Measure Theory and Function Spaces 335

12.1	Measures	335
12.2	Measurable functions	341

12.3	Integration	344
12.4	Convergence theorems	346
12.5	Product measures and Fubini's theorem	348
12.6	The L^p spaces	351
12.7	The basic inequalities	355
12.8	The dual space of L^p	360
12.9	Sobolev spaces	362
12.10	Properties of Sobolev spaces	365
12.11	Laplace's equation	370
12.12	References	373
12.13	Exercises	374
Chapter 13	Differential Calculus and Variational Methods	379
13.1	Linearization	379
13.2	Vector-valued integrals	381
13.3	Derivatives of maps on Banach spaces	386
13.4	The inverse and implicit function theorems	393
13.5	Newton's method	400
13.6	Linearized stability	403
13.7	The calculus of variations	410
13.8	Hamilton's equation and classical mechanics	417
13.9	Multiple integrals in the calculus of variations	420
13.10	References	423
13.11	Exercises	423
	Bibliography	427
	Index	431