Contents

	Pref	face		XXV
1	Elen	nents of p	probability theory ions	1
	1.2	1.2.1 1.2.2	ties of probabilities Joint probabilities Conditional probabilities Bayes' theorem on inverse probabilities	3 3 5 6
	1.3	1.3.1 1.3.2	m variables and probability distributions Transformations of variates Expectations and moments Chebyshev inequality	7 9 11 15
	1.4	1.4.1 1.4.2		16 16 17 19
	1.5	1.5.1 1.5.2 1.5.3 1.5.4 1.5.5 1.5.6	Bernoulli or binomial distributions Bernoulli or binomial distribution Poisson distribution Bose-Einstein distribution The weak law of large numbers Normal or Gaussian distribution The central limit theorem Gamma distribution	21 23 25 26 27 30 30
	1.6	1.6.1 1.6.2 1.6.3	1	33 36 37 38 39
		Probler	ms	37
2	Ran 2.1	2.1.1 2.1.2	stochastic) processes action to statistical ensembles The ensemble average Joint probabilities and correlations The probability functional	41 41 41 43 44

2.2	2 Statio	onarity and ergodicity	45	
	2.2.	1 The time average of a stationary process	47	
	2.2.	2 Ergodicity	48	
		3 Examples of random processes	50	
2.3		erties of the autocorrelation function	52	
2.4	Spect	ral properties of a stationary random process	56	
	2.4.	Spectral density and the Wiener-Khintchine theorem	56	
	۷.4.	2 Singularities of the spectral density	60	
	2.4	Normalized correlations and normalized spectral densities	61	
	2.7.	Closs-correlations and cross-spectral densities	62	
2.5	Ortho	gonal representation of a random process	65	
	2.5.	The Karhunen-Loéve expansion	66	
	2.5.2	The limit $T \to \infty$; an alternative approach to the Wiener-Khintchine theorem	68	
2.6	Time	development and classification of random processes		
	2.6.1	Conditional probability densities	70 70	
	2.6.2	Completely random or separable process	70	
	4.0.3	First-order Markov process	71 72	
	2.6.4	Higher-order Markov process	72 73	
2.7		r equations in integro-differential form	-	
2.8		r equations in differential form	74	
	2.8.1	The Kramers Moved different in	75	
		The Kramers-Moyal differential equation Vector random process	76	
	2.8.3	The order of the Kramers-Moyal differential equation	78	
2.9	Langer	vin equation and Fokker-Planck equation	79 7 0	
	2.9.1	Transition moments for the Langevin process	79	
	2.9.2	Steady-state solution of the Fokker-Planck equation	80	
	2.9.3	Time-dependent solution of the Fokker-Planck equation	81 83	
2.10	The W	iener process (or one-dimensional random walk)	84	
	2.10.1	The random walk problem		
	2.10.2	Joint probabilities and autocorrelation	84	
	2.10.3	Equation of motion of the Wiener process	86 87	
	Probler			
			88	
Some	e ncefni	mathematical to but		
3.1	The co.	mathematical techniques	92	
5.1	2 1 1	nplex analytic signal	92	
	3.1.1	Definition and basic properties of analytic signals	92	
	3.1.2	Quasi-invitociifomatic signals and their envolunce	97	*
	5.1.5	Relationships between correlation functions of real and associated complex analytic random processes	,	
	3.1.4		102	
		Statistical properties of the analytic signal associated with a real Gaussian random process		
3.2	The ano		106	
	3.2.1	rular spectrum representation of wavefields	109	
	3.2.2	The angular spectrum of a wavefield in a slab geometry	109	
	3.2.3	The angular spectrum of a wavefield in a slab geometry An example: diffraction by a semi-transparent object The Weyl representation of	112	
	3.2.4	The Weyl representation of a spherical wave	118	ir
	3.2.5	The Rayleigh diffraction formulas	120	
		- Avridum	125	1

	3.3	The me	thod of stationary phase	128
			Definition of an asymptotic expansion	128
		3.3.2	Method of stationary phase for single integrals	128
			Method of stationary phase for double integrals	133
		3.3.4	An example: far-zone behavior of the angular spectrum	1.41
			representation of wavefields	141
		Problem	ns	144
1	Seco	nd-order	r coherence theory of scalar wavefields	147
	4.1	Introdu	ection	147
	4.2	Some e	lementary concepts and definitions	148
		4.2.1	Temporal coherence and the coherence time	148
		4.2.2	Spatial coherence and the coherence area	150
		4.2.3	Coherence volume and the degeneracy parameter	155
	4.3	Interfer	rence of two stationary light beams as a second-order	
		correlat	tion phenomenon	159
		4.3.1	The laws of interference. The mutual coherence function and the	
			complex degree of coherence	160
		4.3.2	Second-order correlations in the space-frequency domain. The	
			cross-spectral density and the spectral degree of coherence	170
		4.3.3	Coherence time and bandwidth	176
	4.4	Propaga	ation of correlations	180
		4.4.1	Differential equations for the propagation of the mutual coherence	
			and of the cross-spectral density in free space	181
		4.4.2	Propagation of correlations from a plane	183
			Propagation of correlations from finite surfaces	186
		4.4.4	The van Cittert-Zernike theorem	188
		4.4.5	Propagation of correlations from primary sources	193
	4.5	Fields o	of special types	196
			Cross-spectrally pure light	196
			Coherent light in the space-time domain	200
		4.5.3	Coherent light in the space-frequency domain	205
	4.6	Free fie	elds of any state of coherence	207
		4.6.1		
			correlation functions of free fields	208
		4.6.2	Time evolution of the second-order correlation functions of free	211
		4.6.3	fields A relationship between temporal and spatial coherence properties	211
		4.0.3	of free fields	212
		4.6.4	A relationship between spectral properties and spatial coherence	
			properties of free fields	213
	4.7	Cohere	nt-mode representation and ensemble representation of	
			and fields in the space-frequency domain	213
		4.7.1		-
		7./.1	space	214
		4.7.2	Rigorous representation of the cross-spectral density as a	
			correlation function	218
		4.7.3	Natural modes of oscillations of partially coherent primary	
			sources and a representation of their cross-spectral density as a	200
			correlation function	220

		of the pendix of the	4.1 The kernel $H(\mathbf{R})$ of the integral transform representation coperator $\sqrt{(-\nabla^2)}$ [Eq. $(4.6-14)$] 4.2 The Green's function $G(\mathbf{R}, T)$ for the time evolution analytic signal representation of free fields and its Fourier form $\widetilde{G}(\mathbf{R}, \nu)$ [Eqs. $(4.6-19)$ and $(4.6-34)$]	223
			_	224
		Probl	ems	225
5	Rac	diation f	rom sources of any state of coherence	229
	5.1	Introd	uction	229
	5.2		tion from three-dimensional primary sources General formulas	229
		5.2.2	Radiation from some model sources	229
	5.3		tion from planar, secondary sources	233
		5.3.1	General formulas	239
		5.3.2	Radiation from planar, secondary, quasi-homogeneous sources An inverse problem for planar, secondary, quasi-homogeneous	239 242
			oource3	245
	5.4	Equiva	alence theorems for planar sources which generate the same	
		rauran	mensity	250
		5.4.1	An equivalence theorem for planar sources	251
		3.4.2	Example: eduvalent (janssian Scholl model annual	252
		0.4.5	An experimental test of the equivalence theorem	256
	5.55.6		ent-mode representation of Gaussian Schell-model sources	259
	5.0		beams	263
		5.6.2	Monochromatic beams Example: monochromatic Control of the control	263
		5.6.3	Example: monochromatic Gaussian beams Partially coherent beams	267
		5.6.4	Gaussian Schell-model beams	272
	5.7			276
		5.7.1	tions of radiometry	287
		2.7.1	Energy density, energy flux and the energy conservation law in scalar wavefields	
		5.7.2	Basic concepts of radiometry	287
		3.7.3	Radiance function of a planar, secondary, quasi-homogeneous source	292
		5.7.4	Radiative energy transfer model	297
		5.7.5	Radiometry as a short wavelength limit of statistics	301
	50		1 me nomogeneous sources	303
	5.8	radiated		207
		5.8.1	Spectrum of the field generated by two partially correlated sources	307
			quasi-homogeneous sources,	307
			A condition for spectral invariance: the scaling law for planar, secondary, quasi-homogeneous sources	318
	Appe	ndix 5.1	Derivation of the asymptotic approximation approximati	327
4	Appe	ndix 5.2	Product theorem for Gaussian functions	332
		Problem	S Substant functions	335

		Contents	A111	
6		ond-order coherence theory of vector electromagnetic fields	340 340	
	6.1	Introduction	340	
	6.2	The 2×2 equal-time coherence matrix of a well-collimated, uniform, quasi-monochromatic light beam	342	
	6.3	Completely unpolarized and completely polarized light. The degree		
		of polarization	350	
		6.3.1 Unpolarized light (natural light)	350	
		6.3.2 Completely polarized light6.3.3 The degree of polarization	351 352	
	6.4 Transmission of a quasi-monochromatic beam through linear, no			
		image-forming devices	355	
		6.4.1 A compensator	357	
		6.4.2 An absorber	358	
		6.4.3 A rotator	358 359	
		6.4.4 A polarizer 6.4.5 A cascaded system	360	
	4.5	The general second-order coherence matrices and coherence tensors		
	6.5	of a stationary electromagnetic field	363	
		6.5.1 The electric, magnetic and mixed coherence matrices (tensors)	363	
		6.5.2 First-order differential equations for the propagation of the		
		coherence tensors	365	
		6.5.3 Wave equations for propagation of the coherence tensors	367	
	6.6 The second-order cross-spectral density tensors of a stationar			
		electromagnetic field	369	
		6.6.1 The electric, magnetic and mixed cross-spectral density tensors 6.6.2 First-order differential equations for the propagation of the	369 372	
		cross-spectral density tensors 6.6.3 Helmholtz equations for propagation of the cross-spectral density tensors	372	
			373	
		Problems	313	
7	Som	e applications of second-order coherence theory	375	
•	7.1	Introduction	375	
			375	
	7.2	Stellar interferometry		
	7.3	Interference spectroscopy	381	
		7.3.1 General principles	381	
		7.3.2 The phase problem	384	
	7.4	Coherence of transverse laser resonator modes	389	
		7.4.1 Steady-state condition for the cross-spectral density of light at a	200	
		resonator mirror	389 391	
		7.4.2 Nature of the solutions of the integral equation (7.4-7)	371	
	7.5	Dielectric response and the spectrum of induced polarization in a	207	
		fluctuating medium	396	
		7.5.1 Medium whose macroscopic properties do not change in time7.5.2 Medium whose macroscopic properties depend on time in a	396 397	
		deterministic manner 7.5.3 Medium whose macroscopic properties change randomly in time	399	

	7.0	6 Scattering from random media	401	
		7.6.1 Basic equations for deterministic scattering	401	
		7.0.2 Scattering from deterministic media in the first-order Rorn	401	
		approximation 7.6.3 Scattering from random media in the first-order Born	403	
			406	
		7.6.4 Some special cases	406 409	
		7.6.5 Scattering from a simple fluid	414	
	Ap	ppendix 7.1 Evaluation of the expectation value		
		$\langle\langle \mathcal{P}_{1l}^{\uparrow}(\mathbf{k},\omega)\mathcal{P}_{1m}(\mathbf{k}',\omega')\rangle\rangle$ [Eq. (7.6–36)]	418	
		Problems	419	
8	Hio	There and an account of the state of the sta	419	
U	8.1	cher-order correlations in optical fields Introduction	422	
	0.1	miroduction	422	
	8.2	Space-time correlation functions of arbitrary order	422	
	8.3		422	
		a dequation correlation functions of arbitrary order	425	
	8.4	delication randons of fields obeying Gaussian statistics	428	
		0.4.1 Space-time domain	428	
		8.4.2 Space-frequency domain	432	
	8.5	Coherent-mode representation of cross-spectral densities of		
		arollary order	433	
		8.5.1 General expressions	433	
		8.5.2 A single-mode field8.5.3 Fields obeying Gaussian statistics	435	
			436	
		Drobless.	150	
		Problems	436	
9	Sem		436	
9	Sem 9.1	Problems iclassical theory of photoelectric detection of light Introduction	436 438	
9	9.1	niclassical theory of photoelectric detection of light Introduction	436	
9	Sem 9.1 9.2	iclassical theory of photoelectric detection of light	436 438 438	
9	9.1	iclassical theory of photoelectric detection of light Introduction Review of elementary quantum mechanics	436 438 438 439	
9	9.2 9.3	iclassical theory of photoelectric detection of light Introduction Review of elementary quantum mechanics The differential photodetection probability	436 438 438	
9	9.19.29.39.4	iclassical theory of photoelectric detection of light Introduction Review of elementary quantum mechanics	436 438 438 439	
9	9.2 9.3	Introduction Review of elementary quantum mechanics The differential photodetection probability Joint probabilities of multiple photodetections	436 438 438 439 441 445	
9	9.2 9.3 9.4 9.5	Introduction Review of elementary quantum mechanics The differential photodetection probability Joint probabilities of multiple photodetections Integral detection probabilities	436 438 438 439 441	
9	9.19.29.39.4	Introduction Review of elementary quantum mechanics The differential photodetection probability Joint probabilities of multiple photodetections Integral detection probabilities Photoelectric detection in a fluctuating field	436 438 438 439 441 445	
9	9.2 9.3 9.4 9.5 9.6	Introduction Review of elementary quantum mechanics The differential photodetection probability Joint probabilities of multiple photodetections Integral detection probabilities Photoelectric detection in a fluctuating field 9.6.1 Photoelectric bunching	436 438 438 439 441 445 446	
9	9.2 9.3 9.4 9.5	Introduction Review of elementary quantum mechanics The differential photodetection probability Joint probabilities of multiple photodetections Integral detection probabilities Photoelectric detection in a fluctuating field 9.6.1 Photoelectric bunching	436 438 438 439 441 445 446 446 448	
9	9.2 9.3 9.4 9.5 9.6	Introduction Review of elementary quantum mechanics The differential photodetection probability Joint probabilities of multiple photodetections Integral detection probabilities Photoelectric detection in a fluctuating field 9.6.1 Photoelectric bunching Photoelectric counting statistics of a fluctuating field	436 438 438 439 441 445 446 446 448 449	
9	9.2 9.3 9.4 9.5 9.6	Introduction Review of elementary quantum mechanics The differential photodetection probability Joint probabilities of multiple photodetections Integral detection probabilities Photoelectric detection in a fluctuating field 9.6.1 Photoelectric bunching Photoelectric counting statistics of a fluctuating field Photoelectric current fluctuations 9.8.1 Special cases	436 438 438 439 441 445 446 446 448 449 452	
9	9.2 9.3 9.4 9.5 9.6	Introduction Review of elementary quantum mechanics The differential photodetection probability Joint probabilities of multiple photodetections Integral detection probabilities Photoelectric detection in a fluctuating field 9.6.1 Photoelectric bunching Photoelectric counting statistics of a fluctuating field Photoelectric current fluctuations 9.8.1 Special cases 9.8.2 Thermal light	436 438 438 439 441 445 446 446 448 449 452 456	
	9.2 9.3 9.4 9.5 9.6 9.7 9.8	Introduction Review of elementary quantum mechanics The differential photodetection probability Joint probabilities of multiple photodetections Integral detection probabilities Photoelectric detection in a fluctuating field 9.6.1 Photoelectric bunching Photoelectric counting statistics of a fluctuating field Photoelectric current fluctuations 9.8.1 Special cases 9.8.2 Thermal light 9.8.3 Spectral density of the photocurrent	436 438 438 439 441 445 446 446 448 449 452 456 457	
	9.2 9.3 9.4 9.5 9.6 9.7 9.8	Introduction Review of elementary quantum mechanics The differential photodetection probability Joint probabilities of multiple photodetections Integral detection probabilities Photoelectric detection in a fluctuating field 9.6.1 Photoelectric bunching Photoelectric counting statistics of a fluctuating field Photoelectric current fluctuations 9.8.1 Special cases 9.8.2 Thermal light 9.8.3 Spectral density of the photocurrent	436 438 438 439 441 445 446 446 446 448 449 452 456 457 457	
	9.2 9.3 9.4 9.5 9.6 9.7 9.8	Introduction Review of elementary quantum mechanics The differential photodetection probability Joint probabilities of multiple photodetections Integral detection probabilities Photoelectric detection in a fluctuating field 9.6.1 Photoelectric bunching Photoelectric counting statistics of a fluctuating field Photoelectric current fluctuations 9.8.1 Special cases 9.8.2 Thermal light 9.8.3 Spectral density of the photocurrent The Hanbury Brown—Twiss effect (semi-classical treatment)	436 438 438 439 441 445 446 446 448 449 452 456 457	
	9.2 9.3 9.4 9.5 9.6 9.7 9.8	Introduction Review of elementary quantum mechanics The differential photodetection probability Joint probabilities of multiple photodetections Integral detection probabilities Photoelectric detection in a fluctuating field 9.6.1 Photoelectric bunching Photoelectric counting statistics of a fluctuating field Photoelectric current fluctuations 9.8.1 Special cases 9.8.2 Thermal light 9.8.3 Spectral density of the photocurrent The Hanbury Brown—Twiss effect (semi-classical treatment) Stellar intensity interferometry	436 438 438 439 441 445 446 446 446 448 449 452 456 457 457	
	9.2 9.3 9.4 9.5 9.6 9.7 9.8	Introduction Review of elementary quantum mechanics The differential photodetection probability Joint probabilities of multiple photodetections Integral detection probabilities Photoelectric detection in a fluctuating field 9.6.1 Photoelectric bunching Photoelectric counting statistics of a fluctuating field Photoelectric current fluctuations 9.8.1 Special cases 9.8.2 Thermal light 9.8.3 Spectral density of the photocurrent The Hanbury Brown—Twiss effect (semi-classical treatment)	436 438 438 439 441 445 446 446 448 449 452 456 457 457 458 460	
	9.2 9.3 9.4 9.5 9.6 9.7 9.8 9.9 9.10	Introduction Review of elementary quantum mechanics The differential photodetection probability Joint probabilities of multiple photodetections Integral detection probabilities Photoelectric detection in a fluctuating field 9.6.1 Photoelectric bunching Photoelectric counting statistics of a fluctuating field Photoelectric current fluctuations 9.8.1 Special cases 9.8.2 Thermal light 9.8.3 Spectral density of the photocurrent The Hanbury Brown—Twiss effect (semi-classical treatment) Stellar intensity interferometry	436 438 438 439 441 445 446 446 448 449 452 456 457 457 458	

10	O Quantization of the free electromagnetic field 10.1 Introduction		465 465
10.2 Classical field Hamiltonian and the canonical equations of 10.2.1 Plane wave expansions 10.2.2 Unit polarization vectors 10.2.3 Energy of the electromagnetic field		10.2.2 Unit polarization vectors	466 467 468 472
	10.3	Canonical quantization of the transverse field	473
	10.4 Spectrum of the energy; photons 10.4.1 Fock states 10.4.2 Approximately localized photons 10.4.3 Fock states as a basis 10.4.4 The q-representation of the Fock state 10.4.5 Time dependence of the field operators		
	10.5	Momentum of the quantized field	484
10.6 Angular momentum of the quantized field 10.6.1 Angular momentum as a constant of the motion 10.6.2 Decomposition of the total angular momentum 10.6.3 Intrinsic (or spin) angular momentum 10.6.4 Orbital angular momentum		10.6.1 Angular momentum as a constant of the motion10.6.2 Decomposition of the total angular momentum	485 486 488 490 491
	10.7	Phase operators for the quantized field 10.7.1 First attempts to construct a phase operator 10.7.2 Cosine and sine operators 10.7.3 Phase operator based on the phase state projector 10.7.4 Operationally defined phase operators	492 492 494 495 497
	10.8	Space-time commutation relations $10.8.1$ Equations of motion for $\hat{\mathbf{E}}$ and $\hat{\mathbf{B}}$	500 503
	10.9	Vacuum fluctuations 10.9.1 Fluctuations of locally averaged fields 10.9.2 Order of magnitude of vacuum fluctuations 10.9.3 The Casimir force between conductors 10.9.4 The Lamb shift 10.9.5 Vacuum effects in the beam splitter	504 505 507 508 509 511
	10.10	Continuous Fock space	512
	10.11	Some theorems on operator algebra 10.11.1 The operator expansion theorem 10.11.2 Theorems on similarity transformation 10.11.3 Derivative theorems 10.11.4 Normal and antinormal ordering 10.11.5 The Campbell-Baker-Hausdorff theorem	515 515 517 518 518 519
		Problems	520
11	Coher 11.1	rent states of the electromagnetic field Introduction	522 522
	11.2	Fock representation of the coherent state	523
	11.3	The coherent state as a displaced vacuum state – the displacement operator 11.3.1 Properties of the displacement operator	525 526

11	q-representation of the coherent state	528
11		
	11.5.1 Canonical uncertainty product	529 530
	11.5.2 More general states of minimum uncertainty product	530 531
11.	6 Coherent states as a basis; non-orthogonality and over-	551
	completeness	522
	11.6.1 Linear dependence of coherent states	533
	11.0.2 Over-completeness	534 535
	11.6.3 Representation of operators in terms of coherent states	537
11	Evaluation of matrix elements of operators in normal order	537
11.	of states and operators by entire functions	538
11.	brand concrete state representation of the density operator	
	(Glauber-Sudarsnan P-representation)	540
	11.8.1 Quasi-probability densities: the Wissens P. V.	540
	TAILUID A WU VII LIICA OI THE MISMONS FORMANA!	541 542
	11.0.5 Diagolial representation of 6 by a governor of 6	542 543
	11.8.4 Diagonal representation of by a sequence of functions order	J-1J
	11.8.5 Integral representation of $\phi(v)$	544
	11.8.6 Examples of $\phi(v)$	545
11.9		546
	The optical equivalence theorem for normally ordered operators 11.9.1 Quantum characteristic functions	555
11 1	Quantum characteristic functions	557
11.1	0 More general phase space representations	558
	11.10.1 Introduction	558 558
	11.10.2 Operator ordering	
	11.10.3 Application to quantum expectations and the diagonal coherent state representation	
11.1	Multimode fields	561
		562
11.12	11.11.1 Coherent states in the continuous mode representation	565
-1.12	Positive-frequency and negative-frequency field operators 11.12.1 Commutation relations	565
	11.12.1 Commutation relations 11.12.2 Normally ordered correlations	567
11 13	The field produce 11	567
41.10	The field produced by a classical current	568
	Problems	
11 0-	4	570
12 Quan	tum correlations and photon statistics	572
12.1	Introduction	573 573
12.2	Photoelectric measurement of the and the and	573
	Photoelectric measurement of the optical field; normal ordering 12.2.1 Multiple photodetections; birth.	573
	12.2.1 Multiple photodetections; higher-order correlation functions Ordering symbols and ordering operators	576
12.3	Photon density operator	578
12.4	Interference experiments	579
	Interference experiments; second-order correlation functions Correlation functions and group and group and group are a second-order correlation functions.	582
	Correlation functions and cross-spectral densities of arbitrary	J J Z
	•	584
	12.5.2 Cross-spectral densities of a-bis-	585
	12.5.3 Phase-independent phase space functions	588
	r space space functions	589

	Contents	XVII
12	6 Degree and order of coherence	590
12	7 Implications of second-order coherence	593
	12.7.1 Factorization of the correlation functions	
	12.7.2 Correlations of arbitrary order	593 595
	12.7.3 Density operator of the field	595 596
	12.7.4 Wave packets as modes	598
12	, ,genery, isotropy	601
	12.8.1 Stationarity	601
	12.8.2 Condition on the density operator	604
	12.8.3 Properties of cross-spectral densities for stationary fields	605
	12.8.4 Homogeneity	607
	12.8.5 Isotropy	609
12.	The second secon	611
	12.9.1 The quantum counter	612
	12.9.2 Substitution of differential operators	617
	12.9.3 Phase space functional for anti-normally ordered correlations	618
12.	10 Photon statistics	622
	12.10.1 Probabilities	623
	12.10.2 Tests for non-classical states	625
	12.10.3 Moments of \hat{n}	626
	12.10.4 Generating functions for photon numbers in normal and anti-normal order	
		627
12.	1 The problem of localizing photons	629
	12.11.1 Configuration space photon number operator	630
	12.11.2 Commutation relations	630
	12.11.3 Eigenstates of $\hat{n}(\mathcal{V}, t)$ 12.11.4 Photon statistics in a finite vol	633
	12.11.5 Polychromatic photons and non-locality	635 635
12	2 Effect of an attenuator or beam splitter on the quantum field	
12.	12.12.1 Operator relations	639
	12.12.1 Operator relations 12.12.2 Photon correlations	640
	12.12.3 Michelson interferometer	641 643
	12.12.4 Relationship between input and output states for the beam	043
	splitter splitter	644
12.1	3 Effect of a polarizer on the field	647
12.1	4 Einstein locality and photon correlations	648
	12.14.1 The Einstein-Podolsky-Rosen paradox for an entangled two-	
	photon state	649
	12.14.2 Bell's inequality	651
	12.14.3 Clauser-Horne form of Bell's inequality	653
	12.14.4 Experimental confirmation 12.14.5 Non-classical states and Bell inequalities	655
	•	656
	Problems	657
Rad	ation from thermal equilibrium sources	659
13.1	Blackbody radiation	659
	13.1.1 The density operator	659
	13.1.2 Photon statistics	660
	13.1.3 Polarization	663
	13.1.4 Spectral distributions	664
	13.1.5 The diagonal coherent-state representation of $\hat{\rho}$	666

	 13.1.6 Correlation functions of blackbody radiation 13.1.7 Higher-order correlations 13.1.8 Isotropy of blackbody radiation 13.1.9 Intensity fluctuations of blackbody radiation 	667 670 671 672
13.	2 Thermal light	674
13) , ight counts	675
	13.3.1 Intensity fluctuations of a thermal light beam13.3.2 Photon statistics with equal average occupation numbers	676 679
	Problems	681
14 Qua	antum theory of photoelectric detection of light	(02
14.1	Interactions of a quantized electromagnetic field	683
	14.1.1 Perturbative solution in the interaction picture 14.1.2 The electromagnetic interaction between fields and charges	683 684
	11.1.5 Munipolar Hamiltonian	686 689
14.2	photodetection photodetic	691
	14.2.1 Application to the pure coherent state 14.2.2 Evaluation of the probability in the quasi-monochromatic approximation	696
	14.2.3 Evaluation of the electron matrix element 14.2.4 Evaluation of the detection probability for an axially symmetric detector	698 699
14.3		700
14.4	photodetector	702
14.5	probability	703
11.0	The multiple detection probability for an arbitrary initial state of the field	704
14.6	Photoelectric correlations	704
	14.6.1 The Hanbury Brown-Twiss effect (quantum treatment)	707
14 7	1 1.0.2 I notocurrent correlations	708 709
14.7	- and antibulicining	712
	14.7.1 Coincidence detection	713
	14.7.2 Two-time pulse correlation measurements 14.7.3 Antibunching	714
	14.7.4 The time interval distribution of photoelectric pulses	719 720
14.8	Photoelectric counting statistics	723
	14.8.1 The integral detection probability	
	14.8.2 Examples of the detection probability	724 727
140	14.8.3 A natural measure of coherence time	733
14.9	Properties of the detection probability $p(n, t, t + T)$ 14.9.1 Generating functions and the inversion problem	734
	14.5.2 The second moment and sub-poissonion and second	734
	editeration functions from counting statistics	736 738
	Problems	739
15 Intera	action between light and a two-level atom	
15.1	Dynamical variables for a two-level atom	741
	15.1.1 Atomic energy and atomic dipole moment	741
	onergy and atomic dipole moment	744

_			AIX
	15.2	Bloch-representation of the state	746
		15.2.1 Expectation values of spin operators	748
	15.3	Interaction of an atom with a classical field	749
		15.3.1 Bloch equations	750
		15.3.2 Bloch equations in the rotating frame	753
		15.3.3 The Rabi problem	754
		15.3.4 The response of the atom to a light pulse 15.3.5 Hyperbolic secant 2π-pulse	756 758
	15.4	-	138
		treatment	761
		15.4.1 Absorption and emission of photons	762
	15.5	Interaction between an atom and a quantum field –	
		non-perturbative treatment	765
		15.5.1 Heisenberg equations of motion	765
		15.5.2 Approximate solution – the Einstein A-coefficient and the	703
		Lamb shift 15.5.3 Integral equations of motion	769
		15.5.4 Spontaneous emission	771 772
	15.6	Resonance fluorescence	774
		15.6.1 Time development of the light intensity	775
		15.6.2 The atomic scattering cross-section	779
		15.6.3 Spectrum of the fluorescence	781
		15.6.4 Intensity correlation of the fluorescence 15.6.5 Measurements of photon antibunching	785
		15.6.6 Sub-Poissonian photon statistics	787 788
	15.7	Deflection of atoms by light	790
		15.7.1 Momentum transfer after n spontaneous emissions	791
		15.7.2 Momentum transfer by stimulated emission or gradient forces in a strong field	795
	15.8	Cooling and trapping of atoms	799
		15.8.1 Optical molasses	800
		15.8.2 An estimate of achievable low temperature based on energy balance	004
		********	801
		Problems	803
16	Collec	ctive atomic interactions	805
	16.1	Optical free induction decay	805
		16.1.1 Experiments	808
	16.2	Photon echo	809
		16.2.1 Echo experiments	812
	16.3	Self-induced transparency	813
		16.3.1 Equations of motion of the pulse envelope	814
		16.3.2 The area theorem of McCall and Hahn 16.3.3 The pulse shape	816 818
		16.3.4 The pulse velocity	822
	16.4	Optical bistability	822
	10.7	16.4.1 Absorptive bistability in a ring cavity	824

	16.4.2 Dispersive bistability16.4.3 Chaos in optical bistability	830 831
16.5	Collective atomic states and collective dynamical variables	
	16.5.1 Dicke states	832
	16.5.2 Degeneracy of Dicke states	834 839
16.6		
	16.6.1 Dicke superradiance	839
	16.6.2 Cooperative radiation in an atomic product state	840 841
	10.0.5 Time development of superradiance	843
	16.6.4 Some additional complications	847
	16.6.5 More general cooperative radiation	848
	16.6.6 Superradiant classical oscillations	850
16.7	rational concrent states	852
	16.7.1 Bloch-representation of the atomic coherent state	854
	10.7.2 INOH-ORTHOGONALITY and over-completeness •	855
	16.7.3 The atomic state produced by a classical field	857
	Problems	858
17 Some	e general techniques for treating interacting systems	0.00
17.1	The quantum regression theorem	860
	17.1.1 Single-time expectation values	860
	17.1.2 Multi-time expectation values	862
	1/.1.3 Spontaneous atomic emission	862
	1/.1.4 Resonance fluorescence of a two-level atom	863 864
	17.1.5 Quantum regression theorem for normally ordered field	004
15.0	operators	866
17.2	The fluctuation-dissipation theorem	867
	17.2.1 A simple classical linear dissipative system	867
	17.2.2 Quantum mechanical linear dissipative system 17.2.3 Power dissipation	870
	17.2.4 Current fluctuations in thermal equilibrium	871
	17.2.5 Spectral density of the fluctuations	872
	17.2.0 Drownian motion of a particle	874
	17.2.7 Field fluctuations in blackbody radiation	876
17.3	Master equations	876
	17.3.1 The Pauli master equation	877
	1/.3.2 Zwanzig's generalized master equation	878
	The Application to the Pauli equation	880
	17.3.4 Application to the Dicke problem	882 883
17.4	17.3.5 Linear damping of off-diagonal matrix elements	886
17.4	Quantum noise sources and quantum Langevin equations	887
	17:7:1 IIIIIOUUCIIOn	887
	17.4.2 Equations of motion of the quantum system 17.4.3 Commutation relations	888
	17.4.5 Commutation relations 17.4.4 Two-time correlation functions	891
	17.4.3 Langevin equation for the avoice:	893
	17.4.5 Langevin equation for the excitation of the system oscillator 17.4.6 Irreversibility and the arrow of time	896
	Problems	897

	Contents	XX1
18 The	single-mode laser	900
18.	Introduction	900
	18.1.1 Condition for laser action	902
18.2	2 Semiclassical theory of the laser	903
	18.2.1 Normal modes of a cavity	905
	18.2.2 Equation of motion of the laser field	908
	18.2.3 The phase transition analogy	913
18.3	spentaneous emission noise	915
10.4	18.3.1 Fokker-Planck equation	916
	18.3.2 Steady-state solution	918
	18.3.3 The phase transition analogy for a fluctuating laser field	921
	18.3.4 Moments of the light intensity	924
18.4	2	929
	18.4.1 Master equation of the laser field 18.4.2 Photon statistics	931
	18.4.3 Steady-state probability	936
10.5		937
18.5	1 1 I I I I I I I I I I I I I I I I I I	941
	18.5.1 Coherent-state representation of the laser field	941
	18.5.2 Steady-state solution of the master equation	945
18.6	1	945
	18.6.1 Growth of the laser field from the vacuum state	950
	18.6.2 Experimental investigations	953
18.7		954
	18.7.1 Green's functions	956
	18.7.2 Intensity correlation	956
	18.7.3 Field amplitude correlation function and spectral density 18.7.4 Higher-order correlations	959
10.0	-	961
18.8	Laser instabilities and chaos	965
	18.8.1 Relationship to the Lorenz model	967
	18.8.2 Linear stability analysis 18.8.3 Examples of laser instabilities	968
	18.8.4 A test for deterministic chaos	970 972
	Problems	
	Troolems	974
The	two-mode ring laser	976
19.1	Equations of motion	977
	19.1.1 Inclusion of spontaneous emission fluctuations	980
19.2	The steady-state solution	982
	19.2.1 Moments of the light intensity	984
	19.2.2 Comparison with experiment	986
19.3	The phase transition analogy	989
	19.3.1 Minima of the potential	991
	19.3.2 The case coupling constant $\xi < 1$: second-order phase transition	993
	19.3.3 The case coupling constant $\xi > 1$: first-order phase transition	994
	19.3.4 Latent heat of the phase transition	1000

40	4	
19.4	Time-dependent solution when mode coupling $\xi = 1$	1002
	19.4.1 Solution by separation of variables when $\xi = 1$, $a_1 = a_2$ 19.4.2 Green's function	1003
	19.4.3 Correlation functions	1007
19.5		1008
17.5	Time-dependent behavior in the more general case of mode coupling $\xi \neq 1$	
	19.5.1 Mode switching and first passage times	1011
	19.5.2 First passage time distributions	1012
	Problems	1018
		1019
20 The	linear light amplifier	1021
20.1	Introduction	1021
20.2	Master equation for the annulist of the	1021
	Master equation for the amplifier field	1022
20.3	of the master equation	1000
	20.3.1 Input-output correlations	1023
20.4	Photon statistics	1025
	20.4.1 Probability distributions	1025
	20.4.2 The fully inverted light amplifier	1027
20.5	Squeezed light	1029
20.	_	1030
20.6	Condition for the amplifier output field to be classical	1031
	Problems	1032
		1032
1 Squee	ezed states of light	1024
21.1	Definition of quadrature squeezing	1034
21.2		1034
21.3	The unitary squeeze operator	1036
	21.3.1 Squeezing of the two-photon coherent state	1038
	21.3.2 Action of the squeeze operator on any state	1040
21.4	Ideal squeezed states	1040
	21.4.1 Photon statistics	1042
21.5		1044
21.5	Two-photon coherent states	1046
	21.5.1 Transformed Fock states	1040
	21.5.2 Coherent-state representation of the two-photon coherent state 21.5.3 Photon statistics of the two-photon coherent state	1047
21.6	The photon confirm ctata	1049
	Detection of squeezing by homodyning with coherent light	1051
21.7	Squeezing produced in practice: degenerate parametric down-	
		1053
	21.7.1 Production of a squeezed state in degenerate parametric down-conversion	2000
21.8	Broadband squeezed light	1054
	21.8.1 Homodyning and correlation forms	1056
	21.8.2 Quadrature correlations	1058
		1060

		XXIII
	21.8.3 Spectral correlations 21.8.4 Spectral component squeezing and degree of squeezing 21.8.5 Examples of the degree of squeezing $Q(\omega)$	1061 1063 1063
21.9	Higher-order squeezing	1065
	21.9.1 Nth-order squeezing of the two-photon coherent state 21.9.2 Amplitude-squared squeezing	1066 1067
	Problems	1068
Some	e quantum effects in nonlinear optics	1069
22.1	Introduction	1069
22.2	Energy of the field in a dielectric	1069
22.3	Optical harmonic generation	1070
	22.3.1 Squeezing in harmonic generation	1073
22.4	Parametric down-conversion	1074
	22.4.1 Solution of the equations of motion	1075
	22.4.2 Photon statistics	1076
	22.4.3 Proof of non-classical behavior 22.4.4 Multimode perturbative treatment of parametric down-	1078
	conversion	1079
	22.4.5 Entangled quantum state	1082
	22.4.6 Rate of down-conversion	1083
	22.4.7 Time separation between signal and idler photons	1084
	22.4.8 Interference experiments with two down-converters	1088
22.5	Degenerate four-wave mixing	1093
	22.5.1 Equations of motion	1095
	22.5.2 Application to the coherent state	1096
	22.5.3 Squeezing in four-wave mixing 22.5.4 Phase conjugation	1096 1098
22.6	Quantum non-demolition measurements	1100
	22.6.1 The Kerr effect – an example of a QND variable	1102
	22.6.2 Analysis of an interference experiment	1103
	22.6.3 Calculation of fringe visibility	1105
	22.6.4 The probe wave phase shift 22.6.5 Complementarity	1105
	•	1106
Problems		1107
References		1109
Author index		1140
Subject index		1159