

TABLE OF CONTENTS

Preface	ix
1. Random Variables and Generating Functions	1
1.1. Introduction to Random Variables	1
1.2. Expectation	7
1.3. The Characteristic Function	10
1.4. The Laplace Transform	13
1.5. Probability Generating Functions	14
1.6. Cumulants and Cumulant Generating Functions	17
1.7. Probability Weighted Moments	19
1.8. L-Moments	19
1.9. Experimental and Theoretical Moments	20
1.9.1. Gamma Distribution	21
1.9.2. Fickian Distribution	22
1.9.3. Log-Normal Distribution	22
Appendix A: Exponential Distributions	26
Appendix B: Maximum Likelihood Estimation	27
2. Laplace Transforms for Solute Transport Models	29
2.1. Definition of the Transform and its Inverse	29
2.2. Singularities of the Laplace Transform	32
2.3. Green's Functions for Initial Value Problems	34
2.4. Solute Transport by Diffusion	35
2.5. Advective – Dispersive Solute Transport Model	38
2.6. Role of Boundary Conditions	43
2.7. The Mobile – Immobile Water Model	46
2.8. The Physical Nonequilibrium Model	49
2.9. The Chemical Nonequilibrium Model	52
2.10. Nonequilibrium Sorption by Diffusion into Spherical Grains	54
3. Fourier Transforms for Solute Transport Models	57
3.1. Solute Transport by Diffusion	57
3.2. Fourier Transform Pair	60

3.3.	Fourier Transform of the Diffusion Equation	61
3.4.	Fourier Transforms of Derivatives	63
3.5.	Fourier Sine and Cosine Transforms	65
3.6.	Fourier Transform Solution for Advection-Dispersion Equation Over an Infinite Domain	70
3.7.	Fourier Sine Transform for Advection-Dispersion Equation Over Semi-Infinite Domains	71
3.8.	Fourier Transforms in Higher Dimensions	72
4.	Transfer Function Approaches	77
4.1.	Residence Time Distributions	77
4.2.	Models of Solute Transport	78
4.3.	Depth Moments of the Advection-Dispersion Equation	83
4.4.	Depth-Moments for Stochastic-Convective Models	87
4.5.	Transfer Functions for Layered Soils	88
4.6.	Stochastic Stream Tube Models	91
4.7.	A Stochastic Stream Tube Model for Contaminant Dissolution and Transport with Degradation	93
4.7.1.	Local Model	93
5.	Temporal Moment Analysis for Solute Transport in Porous Media	105
5.1.	Model Descriptions and Governing Differential Equations	106
5.2.	Temporal Moment Definitions	109
5.3.	Aris's Method of Moment Analysis	111
5.4.	Computing Time Moments from Experimental Data	116
5.4.1.	Experimental Data	116
5.4.2.	Computing Moments from Observed Data	117
5.4.3.	Estimation Errors	119
5.5.	Applications of the Method of Moments	123
5.5.1.	Estimating Parameters of the Transport Equation	123
5.5.2.	Effective Parameters	128
5.5.3.	Nonequilibrium Indices	129
5.6.	Summary	134
5.7.	Appendix: Sample BTC Data	135
6.	Spatial Moment Analysis for Solute Transport in Porous Media	143
6.1.	Introduction	143
6.2.	Spatial Moments	145
6.3.	Spatial Moments to Describe Solute Plume Behavior	146
6.4.	Spatial Moments for the PNE Model	149
6.5.	Spatial Moments for First-Order Rate Model	151
7.	Moment Analysis for Volatile Compounds	155
7.1.	Introduction	155
7.2.	Immobile Vapor Phase Model	157

7.3.	Description of Loss Fractions	162
7.4.	Effective Parameter Definitions	163
7.5.	Mobile Vapor Phase Model	173
7.6.	Spatial Moments for Mobile Vapor Phase Model	181
8.	Moment Generating Differential Equations	183
8.1.	Definitions of MGDEs	183
8.2.	Temporal MGDEs for Solute Transport in Soil	185
8.2.1.	Analysis with Degradation	186
8.2.2.	Analysis without Degradation	187
8.3.	Spatial MGDEs for PNE Model of Solute Transport in Soil	189
8.3.1.	Zeroth Moment	192
8.3.2.	First Moment	197
8.3.3.	Second Moment	198
8.4.	Spatial Moments for a Two-Layer Aquifer	199
8.5.	Perfectly Stratified Aquifer with Velocity Variation	203
9.	Moment Analysis for Compounds Undergoing Sequential Decay Chain Reactions	207
9.1.	Introduction	207
9.2.	Governing Differential Equations	207
9.3.	Laplace Transforms	209
9.4.	Temporal Moment Analysis	210
9.5.	Temporal Moments for Advective Transport	215
9.6.	Spatial Moments for Compounds Undergoing Sequential First-Order Decay Chain	217
10.	Applications of Moments in Interval Computing Methods	223
10.1.	General Remarks	223
10.2.	Interval Arithmetic Operations	224
10.3.	Interval Distribution Functions	225
10.4.	Defining Moments from Interval Distribution Functions	227
10.5.	Application to a Remediation Example	227
10.5.1.	First-Order Degradation Model	228
10.5.2.	Statistical Distributions of Degradation Rates and Initial Concentrations	229
10.5.3.	Field-Scale Models	231
10.5.4.	Results and Discussion	233
10.6.	Application to Solute Transport Experiment	237
10.6.1.	Description of the Solute Transport Experiment	238
10.6.2.	Advective Solute Transport in Vadose Zone	238
10.6.3.	Field-Scale Model using Interval Computing Method	240
10.6.4.	Stochastic Advective Solute Transport	241
10.6.5.	Results and Discussion	244

11. Moment Analysis for Subsurface Storm Flow	247
11.1. Introduction	247
11.2. Moments For Linearized Subsurface Drainage with No Recharge	249
11.3. Subsurface Drainage with Lateral Inflow	252
11.4. Transfer Function Approach for Subsurface Drainage	256
11.4.1. Theoretical Development	256
11.4.2. Moments and Experimental Results	260
12. Constructing Concentration Distributions from Moments	265
12.1. Problem Definition	265
12.2. Density Matching Methods	267
12.3. Polynomial Summation Methods	267
12.3.1. Gram-Charlier and Edgeworth Series Approach	267
12.3.2. Expansion Methods Based on other Polynomials	270
12.4. Maximum Entropy (Maxent) Method	271
12.4.1. Geometrical Moments	273
12.5. Example Calculations	274
References	277
Index	289