

Contents

	<i>Preface</i>	page xi
1	Introduction	1
2	Algebraic preliminaries	13
2.1	The classes $\mathcal{P}_{const}(J)$ and $\mathcal{U}_{const}(J)$	13
2.2	The Potapov–Ginzburg transform	14
2.3	Linear fractional transformations	24
2.4	Matrix balls	30
2.5	Redheffer transforms	34
2.6	Indefinite metrics in the space \mathbb{C}^m	37
2.7	Minus matrices	45
2.8	Strictly minus matrices	48
2.9	Linear fractional transformations in $\mathcal{S}_{const}^{p \times q}$	58
2.10	Linear fractional transformations in $\mathcal{C}_{const}^{p \times p}$	63
2.11	Transformations from $\mathcal{S}_{const}^{p \times p}$ into $\mathcal{C}_{const}^{p \times p}$	66
2.12	Affine generalizations	68
2.13	The J modulus	72
2.14	Infinite products of matrices	77
2.15	Some useful inequalities	80
2.16	Bibliographical notes	81
3	The Nevanlinna class of meromorphic mvf's	83
3.1	Basic classes of functions	83
3.2	The Riesz-Herglotz-Nevanlinna representation	87
3.3	Some subclasses of the Carathéodory class $\mathcal{C}^{p \times p}$	91
3.4	Inner-outer factorization in the class H_∞	93
3.5	Factorization in the classes \mathcal{N}_+ and \mathcal{N}_-	98
3.6	The rank of meromorphic mvf's	103
3.7	Inner and outer mvf's in $H_\infty^{p \times q}$	104

3.8	Fourier transforms and Paley-Wiener theorems	105
3.9	The Beurling-Lax theorem	107
3.10	Inner-outer factorization in $H_\infty^{p \times q}$	114
3.11	The Smirnov maximum principle	118
3.12	Characterization via multiplication operators	122
3.13	Factorization in $\mathcal{N}^{p \times q}$ and denominators	126
3.14	Some properties of outer mvf's	133
3.15	Scalar denominators	136
3.16	Factorization of positive semidefinite mvf's	138
3.17	Blaschke-Potapov products in $\mathcal{S}^{p \times p}$	142
3.18	Entire matrix functions of class $\mathcal{N}^{p \times q}$	147
3.19	Entire inner mvf's	151
3.20	Minimal denominators of mvf's $f \in \mathcal{E} \cap \mathcal{N}^{p \times q}$	155
3.21	The class $\Pi^{p \times q}$	157
3.22	Mean types for mvf's in $\mathcal{N}^{p \times q}$ and $\Pi^{p \times q}$	163
3.23	Bibliographical notes	166
4	<i>J</i>-contractive and <i>J</i>-inner matrix valued functions	169
4.1	The classes $\mathcal{P}(J)$ and $\mathcal{U}(J)$	169
4.2	Blaschke-Potapov products	174
4.3	Entire <i>J</i> -inner mvf's	181
4.4	Multiplicative representations in the class $\mathcal{P}^\circ(J)$	185
4.5	The classes $\mathcal{P}(j_{pq})$ and $\mathcal{U}(j_{pq})$	187
4.6	Associated pairs of the first kind	190
4.7	The classes $\mathcal{P}(J_p)$, $\mathcal{P}(j_p, J_p)$, $\mathcal{U}(J_p)$ and $\mathcal{U}(j_p, J_p)$	195
4.8	Associated pairs of the second kind	198
4.9	Singular and regular <i>J</i> -inner mvf's	201
4.10	Domains of holomorphy of <i>J</i> inner mvf's	207
4.11	Growth constraints on <i>J</i> -inner mvf's	208
4.12	Monotonic sequences in $\mathcal{P}^\circ(J)$	209
4.13	Linear fractional transformations	211
4.14	Connections between the two kinds of associated pairs	217
4.15	Redheffer transforms	218
4.16	Strongly regular <i>J</i> -inner mvf's	219
4.17	Minus matrix valued functions	221
4.18	More on linear fractional transformations	230
4.19	Bibliographical notes	237
5	Reproducing kernel Hilbert spaces	240
5.1	Preliminaries	240
5.2	Examples	247

5.3	A characterization of the spaces $\mathcal{H}(U)$	254
5.4	A finite dimensional example and extensions thereof	261
5.5	Extension of $\mathcal{H}(U)$ into \mathbb{C} for $U \in \mathcal{P}^\circ(J)$	266
5.6	The space $\mathcal{H}(b)$ for $p \times p$ inner mvf's $b(\lambda)$	270
5.7	The space $\mathcal{H}_*(b)$ for $p \times p$ inner mvf's $b(\lambda)$	275
5.8	The space $\mathcal{H}(U)$ for $m \times m$ J -inner mvf's $U(\lambda)$	276
5.9	The space $\mathcal{H}(W)$ for $W \in \mathcal{U}(j_{pq})$	289
5.10	The de Branges space $\mathcal{B}(\mathfrak{E})$	293
5.11	Regular de Branges matrices \mathfrak{E} and spaces $\mathcal{B}(\mathfrak{E})$	298
5.12	Connections between A and \mathfrak{E}	300
5.13	A factorization and parametrization of mvf's $A \in \mathcal{U}(J_p)$.	302
5.14	A description of $\mathcal{H}(W) \cap L_2^m$	308
5.15	Characterizations of the classes $\mathcal{U}_S(J)$, $\mathcal{U}_{rR}(J)$ and $\mathcal{U}_{rsR}(J)$	313
5.16	Regular J -inner mvf's	318
5.17	Formulas for $W(\lambda)$ when $W \in \mathcal{U}_{rsR}(j_{pq})$	318
5.18	A description of $\mathcal{H}(A)$ when $A \in \mathcal{U}_{rsR}(J_p)$	322
5.19	Bibliographical notes	332
6	Operator nodes and passive systems	334
6.1	Characteristic mvf's of Livsic-Brodskii nodes	334
6.2	Connections with systems theory	348
6.3	Bibliographical notes	354
7	Generalized interpolation problems	358
7.1	The Nehari problem	359
7.2	γ -generating matrices	364
7.3	Criteria for right regularity	381
7.4	Criteria for left regularity	390
7.5	Formulas for \mathfrak{A} when $\ \Gamma\ < 1$	391
7.6	The generalized Schur interpolation problem	396
7.7	Generalized Sarason problems in $S^{p \times q}$ and $H_\infty^{p \times q}$	401
7.8	Resolvent matrices for the GSP in the Schur class	404
7.9	The generalized Carathéodory interpolation problem	408
7.10	Connections between the GSIP and the GCIP	410
7.11	Generalized Sarason problems in $\mathring{C}^{p \times p}$	413
7.12	Bibliographical notes	417
8	Generalized Krein extension problems	420
8.1	Helical extension problems	420
8.2	Positive definite extension problems	439
8.3	Positive definite and helical extension problems	441
8.4	Resolvent matrices for positive definite extension problems	443

8.5	Tangential and bitangential positive extension problems	446
8.6	The Krein accelerant extension problem	449
8.7	The accelerant and helical extension problems	451
8.8	Conditions for strictly completely indeterminate HEP($g; a$)	452
8.9	Formulas for the normalized resolvent matrix for the HEP	457
8.10	B -resolvent matrices for the accelerant case	460
8.11	Bibliographical notes	467
9	Darlington representations and related inverse problems	470
9.1	\mathcal{D} -representations of mvf's $s \in \Pi \cap \mathcal{S}^{p \times q}$	471
9.2	Chain scattering \mathcal{D} -representations	481
9.3	\mathcal{D} -representations in the class $\mathcal{C}^{p \times p}$	484
9.4	de Branges matrices and spectral functions	486
9.5	The inverse 12 block problem for j_{pq} -inner mvf's	489
9.6	Inverse problems in special classes	490
9.7	$J_{p,r}$ -inner SI-dilations of $c \in \Pi \cap \mathcal{C}^{p \times p}$	499
9.8	Rational approximation of mvf's of the class $\Pi \cap H_\infty^{p \times q}$	506
9.9	Bibliographical notes	512
10	More criteria for strong regularity	515
10.1	The matrix Muckenhoupt (A_2) condition	515
10.2	Criteria of strong regularity for γ -generating matrices	518
10.3	Strong regularity for J-inner mvf's	521
10.4	A mvf $U \in \mathcal{E} \cap \mathcal{U}_{rsR}(J_1)$ that is not in $L_\infty^{2 \times 2}$	525
10.5	Right and left strongly regular de Branges matrices	528
10.6	LB J -nodes with characteristic mvf's in $\mathcal{U}_{rsR}(J)$	531
10.7	Bibliographical notes	533
11	Formulas for entropy functionals	536
11.1	Definitions of the entropy functionals	536
11.2	Motivation for the terminology entropy functional	537
11.3	Entropy of completely indeterminate interpolation problems	538
11.4	Formulas for entropy functionals and their minimum	540
11.5	A matricial generalization of the entropy inequality	546
11.6	Bibliographical notes	549
<i>Bibliography</i>		551
<i>Notation index</i>		567
<i>Subject index</i>		571