

Contents

List of Figures	xxi
Preface	xxxvii
Chapter 1	
Introduction	1
1.1 Content Overview	2
1.1.1 Chapter 2: Collision Detection Design Issues	2
1.1.2 Chapter 3: A Math and Geometry Primer	2
1.1.3 Chapter 4: Bounding Volumes	3
1.1.4 Chapter 5: Basic Primitive Tests	3
1.1.5 Chapter 6: Bounding Volume Hierarchies	3
1.1.6 Chapter 7: Spatial Partitioning	3
1.1.7 Chapter 8: BSP Tree Hierarchies	4
1.1.8 Chapter 9: Convexity-based Methods	4
1.1.9 Chapter 10: GPU-assisted Collision Detection	4
1.1.10 Chapter 11: Numerical Robustness	4
1.1.11 Chapter 12: Geometrical Robustness	4
1.1.12 Chapter 13: Optimization	5
1.2 About the Code	5
Chapter 2	
Collision Detection Design Issues	7
2.1 Collision Algorithm Design Factors	7
2.2 Application Domain Representation	8
2.2.1 Object Representations	8
2.2.2 Collision Versus Rendering Geometry	11
2.2.3 Collision Algorithm Specialization	12
2.3 Types of Queries	13
2.4 Environment Simulation Parameters	14

2.4.1	Number of Objects	14
2.4.2	Sequential Versus Simultaneous Motion	15
2.4.3	Discrete Versus Continuous Motion	16
2.5	Performance	17
2.5.1	Optimization Overview	18
2.6	Robustness	19
2.7	Ease of Implementation and Use	19
2.7.1	Debugging a Collision Detection System	20
2.8	Summary	21

Chapter 3

A Math and Geometry Primer **23**

3.1	Matrices	23
3.1.1	Matrix Arithmetic	25
3.1.2	Algebraic Identities Involving Matrices	26
3.1.3	Determinants	27
3.1.4	Solving Small Systems of Linear Equations Using Cramer's Rule	29
3.1.5	Matrix Inverses for 2×2 and 3×3 Matrices	31
3.1.6	Determinant Predicates	32
3.1.6.1	ORIENT2D(A, B, C)	32
3.1.6.2	ORIENT3D(A, B, C, D)	33
3.1.6.3	INCIRCLE2D(A, B, C, D)	34
3.1.6.4	INSPHERE(A, B, C, D, E)	34
3.2	Coordinate Systems and Points	35
3.3	Vectors	35
3.3.1	Vector Arithmetic	37
3.3.2	Algebraic Identities Involving Vectors	38
3.3.3	The Dot Product	39
3.3.4	Algebraic Identities Involving Dot Products	40
3.3.5	The Cross Product	41
3.3.6	Algebraic Identities Involving Cross Products	44
3.3.7	The Scalar Triple Product	44
3.3.8	Algebraic Identities Involving Scalar Triple Products	46
3.4	Barycentric Coordinates	46
3.5	Lines, Rays, and Segments	53
3.6	Planes and Halfspaces	54

3.7	Polygons	56
3.7.1	Testing Polygonal Convexity	59
3.8	Polyhedra	62
3.8.1	Testing Polyhedral Convexity	64
3.9	Computing Convex Hulls	64
3.9.1	Andrew's Algorithm	65
3.9.2	The Quickhull Algorithm	66
3.10	Voronoi Regions	69
3.11	Minkowski Sum and Difference	70
3.12	Summary	72

Chapter 4

Bounding Volumes	75	
4.1	Desirable BV Characteristics	76
4.2	Axis-aligned Bounding Boxes (AABBs)	77
4.2.1	AABB-AABB Intersection	79
4.2.2	Computing and Updating AABBs	81
4.2.3	AABB from the Object Bounding Sphere	82
4.2.4	AABB Reconstructed from Original Point Set	82
4.2.5	AABB from Hill-climbing Vertices of the Object Representation	84
4.2.6	AABB Recomputed from Rotated AABB	86
4.3	Spheres	88
4.3.1	Sphere-sphere Intersection	88
4.3.2	Computing a Bounding Sphere	89
4.3.3	Bounding Sphere from Direction of Maximum Spread	91
4.3.4	Bounding Sphere Through Iterative Refinement	98
4.3.5	The Minimum Bounding Sphere	99
4.4	Oriented Bounding Boxes (OBBs)	101
4.4.1	OBB-OBB Intersection	101
4.4.2	Making the Separating-axis Test Robust	106
4.4.3	Computing a Tight OBB	107
4.4.4	Optimizing PCA-based OBBs	109
4.4.5	Brute-force OBB Fitting	112
4.5	Sphere-swept Volumes	112
4.5.1	Sphere-swept Volume Intersection	114
4.5.2	Computing Sphere-swept Bounding Volumes	115

4.6	Halfspace Intersection Volumes	115
4.6.1	Kay–Kajiya Slab-based Volumes	116
4.6.2	Discrete-orientation Polytopes (k -DOPs)	117
4.6.3	k -DOP– k -DOP Overlap Test	118
4.6.4	Computing and Realigning k -DOPs	119
4.6.5	Approximate Convex Hull Intersection Tests	121
4.7	Other Bounding Volumes	122
4.8	Summary	123

Chapter 5

Basic Primitive Tests 125

5.1	Closest-point Computations	125
5.1.1	Closest Point on Plane to Point	126
5.1.2	Closest Point on Line Segment to Point	127
5.1.2.1	Distance of Point To Segment	129
5.1.3	Closest Point on AABB to Point	130
5.1.3.1	Distance of Point to AABB	131
5.1.4	Closest Point on OBB to Point	132
5.1.4.1	Distance of Point to OBB	134
5.1.4.2	Closest Point on 3D Rectangle to Point	135
5.1.5	Closest Point on Triangle to Point	136
5.1.6	Closest Point on Tetrahedron to Point	142
5.1.7	Closest Point on Convex Polyhedron to Point	145
5.1.8	Closest Points of Two Lines	146
5.1.9	Closest Points of Two Line Segments	148
5.1.9.1	2D Segment Intersection	151
5.1.10	Closest Points of a Line Segment and a Triangle	153
5.1.11	Closest Points of Two Triangles	155
5.2	Testing Primitives	156
5.2.1	Separating-axis Test	156
5.2.1.1	Robustness of the Separating-axis Test	159
5.2.2	Testing Sphere Against Plane	160
5.2.3	Testing Box Against Plane	161
5.2.4	Testing Cone Against Plane	164
5.2.5	Testing Sphere Against AABB	165

5.2.6	Testing Sphere Against OBB	166
5.2.7	Testing Sphere Against Triangle	167
5.2.8	Testing Sphere Against Polygon	168
5.2.9	Testing AABB Against Triangle	169
5.2.10	Testing Triangle Against Triangle	172
5.3	Intersecting Lines, Rays, and (Directed) Segments	175
5.3.1	Intersecting Segment Against Plane	175
5.3.2	Intersecting Ray or Segment Against Sphere	177
5.3.3	Intersecting Ray or Segment Against Box	179
5.3.4	Intersecting Line Against Triangle	184
5.3.5	Intersecting Line Against Quadrilateral	188
5.3.6	Intersecting Ray or Segment Against Triangle	190
5.3.7	Intersecting Ray or Segment Against Cylinder	194
5.3.8	Intersecting Ray or Segment Against Convex Polyhedron	198
5.4	Additional Tests	201
5.4.1	Testing Point in Polygon	201
5.4.2	Testing Point in Triangle	203
5.4.3	Testing Point in Polyhedron	206
5.4.4	Intersection of Two Planes	207
5.4.5	Intersection of Three Planes	211
5.5	Dynamic Intersection Tests	214
5.5.1	Interval Halving for Intersecting Moving Objects	215
5.5.2	Separating Axis Test for Moving Convex Objects	219
5.5.3	Intersecting Moving Sphere Against Plane	219
5.5.4	Intersecting Moving AABB Against Plane	222
5.5.5	Intersecting Moving Sphere Against Sphere	223
5.5.6	Intersecting Moving Sphere Against Triangle (and Polygon)	226
5.5.7	Intersecting Moving Sphere Against AABB	228
5.5.8	Intersecting Moving AABB Against AABB	230
5.6	Summary	232

Chapter 6

Bounding Volume Hierarchies	235	
6.1	Hierarchy Design Issues	236
6.1.1	Desired BVH Characteristics	236

6.1.2	Cost Functions	237
6.1.3	Tree Degree	238
6.2	Building Strategies for Hierarchy Construction	239
6.2.1	Top-down Construction	240
6.2.1.1	Partitioning Strategies	241
6.2.1.2	Choice of Partitioning Axis	243
6.2.1.3	Choice of Split Point	244
6.2.2	Bottom-up Construction	245
6.2.2.1	Improved Bottom-up Construction	247
6.2.2.2	Other Bottom-up Construction Strategies	249
6.2.2.3	Bottom-up n -ary Clustering Trees	250
6.2.3	Incremental (Insertion) Construction	251
6.2.3.1	The Goldsmith–Salmon Incremental Construction Method	252
6.3	Hierarchy Traversal	253
6.3.1	Descent Rules	254
6.3.2	Generic Informed Depth-first Traversal	256
6.3.3	Simultaneous Depth-first Traversal	259
6.3.4	Optimized Leaf-direct Depth-first Traversal	260
6.4	Sample Bounding Volume Hierarchies	261
6.4.1	OBB Trees	261
6.4.2	AABB Trees and BoxTrees	262
6.4.3	Sphere Tree Through Octree Subdivision	263
6.4.4	Sphere Tree from Sphere-covered Surfaces	264
6.4.5	Generate-and-Prune Sphere Covering	264
6.4.6	k -dop Trees	265
6.5	Merging Bounding Volumes	266
6.5.1	Merging Two AABBs	267
6.5.2	Merging Two Spheres	267
6.5.3	Merging Two OBBs	269
6.5.4	Merging Two k -DOPs	269
6.6	Efficient Tree Representation and Traversal	270
6.6.1	Array Representation	270
6.6.2	Preorder Traversal Order	272
6.6.3	Offsets Instead of Pointers	273

6.6.4	Cache-friendlier Structures (Nonbinary Trees)	274
6.6.5	Tree Node and Primitive Ordering	275
6.6.6	On Recursion	276
6.6.7	Grouping Queries	278
6.7	Improved Queries Through Caching	280
6.7.1	Surface Caching: Caching Intersecting Primitives	280
6.7.2	Front Tracking	282
6.8	Summary	284

Chapter 7

Spatial Partitioning **285**

7.1	Uniform Grids	285
7.1.1	Cell Size Issues	286
7.1.2	Grids as Arrays of Linked Lists	287
7.1.3	Hashed Storage and Infinite Grids	288
7.1.4	Storing Static Data	290
7.1.5	Implicit Grids	291
7.1.6	Uniform Grid Object-Object Test	294
7.1.6.1	One Test at a Time	295
7.1.6.2	All Tests at a Time	297
7.1.7	Additional Grid Considerations	299
7.2	Hierarchical Grids	300
7.2.1	Basic Hgrid Implementation	302
7.2.2	Alternative Hierarchical Grid Representations	306
7.2.3	Other Hierarchical Grids	307
7.3	Trees	307
7.3.1	Octrees (and Quadtrees)	308
7.3.2	Octree Object Assignment	309
7.3.3	Locational Codes and Finding the Octant for a Point	313
7.3.4	Linear Octrees (Hash-based)	314
7.3.5	Computing the Morton Key	316
7.3.6	Loose Octrees	318
7.3.7	k -d Trees	319
7.3.8	Hybrid Schemes	321
7.4	Ray and Directed Line Segment Traversals	322

7.4.1	<i>k-d</i> Tree Intersection Test	322
7.4.2	Uniform Grid Intersection Test	324
7.5	Sort and Sweep Methods	329
7.5.1	Sorted Linked-list Implementation	330
7.5.2	Array-based Sorting	336
7.6	Cells and Portals	338
7.7	Avoiding Retesting	341
7.7.1	Bit Flags	341
7.7.2	Time Stamping	342
7.7.3	Amortized Time Stamp Clearing	344
7.8	Summary	346

Chapter 8

BSP Tree Hierarchies	349	
8.1	BSP Trees	349
8.2	Types of BSP Trees	351
8.2.1	Node-storing BSP Trees	351
8.2.2	Leaf-storing BSP Trees	352
8.2.3	Solid-leaf BSP Trees	354
8.3	Building the BSP Tree	355
8.3.1	Selecting Dividing Planes	358
8.3.2	Evaluating Dividing Planes	361
8.3.3	Classifying Polygons with Respect to a Plane	364
8.3.4	Splitting Polygons Against a Plane	367
8.3.5	More on Polygon Splitting Robustness	372
8.3.6	Tuning BSP Tree Performance	373
8.4	Using the BSP Tree	374
8.4.1	Testing a Point Against a Solid-leaf BSP Tree	374
8.4.2	Intersecting a Ray Against a Solid-leaf BSP Tree	376
8.4.3	Polytope Queries on Solid-leaf BSP Trees	378
8.5	Summary	381

Chapter 9

Convexity-based Methods	383	
9.1	Boundary-based Collision Detection	383

9.2	Closest-features Algorithms	385
9.2.1	The V-Clip Algorithm	386
9.3	Hierarchical Polyhedron Representations	388
9.3.1	The Dobkin–Kirkpatrick Hierarchy	389
9.4	Linear and Quadratic Programming	391
9.4.1	Linear Programming	391
9.4.1.1	Fourier–Motzkin Elimination	394
9.4.1.2	Seidel’s Algorithm	396
9.4.2	Quadratic Programming	398
9.5	The Gilbert–Johnson–Keerthi Algorithm	399
9.5.1	The Gilbert–Johnson–Keerthi Algorithm	400
9.5.2	Finding the Point of Minimum Norm in a Simplex	403
9.5.3	GJK, Closest Points and Contact Manifolds	405
9.5.4	Hill Climbing for Extreme Vertices	405
9.5.5	Exploiting Coherence by Vertex Caching	407
9.5.6	Rotated Objects Optimization	408
9.5.7	GJK for Moving Objects	408
9.6	The Chung–Wang Separating-vector Algorithm	410
9.7	Summary	412

Chapter 10

GPU-assisted Collision Detection **413**

10.1	Interfacing with the GPU	414
10.1.1	Buffer Readbacks	414
10.1.2	Occlusion Queries	416
10.2	Testing Convex Objects	416
10.3	Testing Concave Objects	420
10.4	GPU-based Collision Filtering	423
10.5	Summary	426

Chapter 11

Numerical Robustness **427**

11.1	Robustness Problem Types	427
11.2	Representing Real Numbers	429
11.2.1	The IEEE-754 Floating-point Formats	431

11.2.2	Infinity Arithmetic	435
11.2.3	Floating-point Error Sources	438
11.3	Robust Floating-point Usage	441
11.3.1	Tolerance Comparisons for Floating-point Values	441
11.3.2	Robustness Through Thick Planes	444
11.3.3	Robustness Through Sharing of Calculations	446
11.3.4	Robustness of Fat Objects	448
11.4	Interval Arithmetic	448
11.4.1	Interval Arithmetic Examples	450
11.4.2	Interval Arithmetic in Collision Detection	451
11.5	Exact and Semi-exact Computation	452
11.5.1	Exact Arithmetic Using Integers	453
11.5.2	Integer Division	457
11.5.3	Segment Intersection Using Integer Arithmetic	459
11.6	Further Suggestions for Improving Robustness	462
11.7	Summary	463

Chapter 12

Geometrical Robustness **465**

12.1	Vertex Welding	466
12.2	Computing Adjacency Information	474
12.2.1	Computing a Vertex-to-Face Table	477
12.2.2	Computing an Edge-to-Face Table	479
12.2.3	Testing Connectedness	482
12.3	Holes, Cracks, Gaps and T-Junctions	484
12.4	Merging Co-planar Faces	487
12.4.1	Testing Co-planarity of Two Polygons	489
12.4.2	Testing Polygon Planarity	491
12.5	Triangulation and Convex Partitioning	495
12.5.1	Triangulation by Ear Cutting	496
12.5.1.1	Triangulating Polygons with Holes	499
12.5.2	Convex Decomposition of Polygons	500
12.5.3	Convex Decomposition of Polyhedra	502
12.5.4	Dealing with “Nondecomposable” Concave Geometry	506

12.6	Consistency Testing Using Euler's Formula	507
12.7	Summary	510
Chapter 13		
Optimization		511
13.1	CPU Caches	513
13.2	Instruction Cache Optimizations	515
13.3	Data Cache Optimizations	517
13.3.1	Structure Optimizations	518
13.3.2	Quantized and Compressed Vertex Data	522
13.3.3	Prefetching and Preloading	523
13.4	Cache-aware Data Structures and Algorithms	525
13.4.1	A Compact Static k -d Tree	525
13.4.2	A Compact AABB Tree	529
13.4.3	Cache Obliviousness	530
13.5	Software Caching	531
13.5.1	Cached Linearization Example	532
13.5.2	Amortized Predictive Linearization Caching	535
13.6	Aliasing	536
13.6.1	Type-based Alias Analysis	538
13.6.2	Restricted Pointers	540
13.6.3	Avoiding Aliasing	542
13.7	Parallelism Through SIMD Optimizations	543
13.7.1	Four Spheres Versus Four Spheres SIMD Test	545
13.7.2	Four Spheres Versus Four AABBs SIMD Test	546
13.7.3	Four AABBs Versus Four AABBs SIMD Test	546
13.8	Branching	547
13.9	Summary	551
References		553
Index		577
About the CD ROM		591