

Contents

Introduction – The Theory of Orbits from Epicycles to “Chaos”	1
Chapter 1. Dynamics and Dynamical Systems – Quod Satis	15
<i>A. Dynamical Systems and Newtonian Dynamics</i>	16
1.1 Dynamical Systems: Generalities	16
1.2 Classification of Critical Points – Stability	20
1.3 The n -Dimensional Oscillator	26
<i>B. Lagrangian Dynamics</i>	32
1.4 Lagrange’s Equations	32
1.5 Ignorable Variables and Integration of Lagrange’s Equations	37
1.6 Noether’s Theorem	43
1.7 An Application of Noether’s Theorem: The n -Dimensional Oscillator	50
1.8 The Principle of Least Action in Jacobi Form	57
<i>C. Hamiltonian Dynamics and Hamilton–Jacobi Theory</i>	61
1.9 The Canonical Equations	61
1.10 The Integral Invariants – Liouville’s Theorem	65
1.11 Poisson Brackets and Poisson’s Theorem – The Generation of New Integrals	73
1.12 Canonical Transformations	76
1.13 Generating Functions – Infinitesimal Canonical Transformations	82
1.14 The Extended Phase Space	85
1.15 The Hamilton–Jacobi Equation and the Problem of Separability	90
1.16 Action–Angle Variables	98
1.17 Separable Multiperiodic Systems – Uniqueness of the Action–Angle Variables	106
1.18 Integrals in Involution – Liouville’s Theorem for Integrable Systems	113
1.19 Lax’s Method – The Painlevé Property	117

Chapter 2. The Two-Body Problem	125
2.1 The Two-Body Problem and Kepler's Three Laws	126
2.2 The Laplace–Runge–Lenz Vector	136
2.3 Bertrand's Theorem and Related Questions	141
2.4 The Position of the Point on the Orbit	147
2.5 The Elements of the Orbit	156
2.6 The Problem of Regularization	162
2.7 Topology of the Two-Body Problem	171
Chapter 3. The N-Body Problem	177
3.1 Equations of Motion and the Existence Theorem	178
3.2 The Integrals of the Motion	184
3.3 The Singularities	192
3.4 Sundman's Theorem	198
3.5 The Evolution of the System for $t \rightarrow \infty$	200
3.6 The Virial Theorem	209
3.7 Particular Solutions of the N -Body Problem	216
3.8 Homographic Motions and Central Configurations	229
Chapter 4. The Three-Body Problem	237
4.1 The General Three-Body Problem	238
4.2 Existence of the Solution – Sundman and Levi-Civita Regularization	244
4.3 The Restricted Three-Body Problem	256
4.4 The Stability of the Equilibrium Solutions	265
4.5 The Delaunay Elements for the Restricted Three-Body Problem	272
4.6 The Regularization of the Restricted Three-Body Problem	279
4.7 Extensions and Generalizations of the Restricted Problem	284
Chapter 5. Orbits in Given Potentials	301
5.1 Introduction	302
5.2 Orbits in Spherically Symmetric Potentials	306
5.3 Orbits in Isochronal Potentials	316
5.4 Elliptical Coordinates and Stäckel's Theorem	323
5.5 Planar Potentials	334
5.6 The Problem of Two Fixed Centres in the Plane	341
5.7 Axially Symmetric Potentials – Motion in the Potential of the Earth	349
5.8 Orbits in Triaxial Potentials	352
5.9 Configurational Invariants	359
Mathematical Appendix	363
A.1 Spherical Trigonometry	364

A.2 Curvilinear Coordinate Systems	365
A.3 Riemannian Geometry	370
Bibliographical Notes	375
Name Index	385
Subject Index	389