

CONTENTS

Preface	vi
Chapter 1. Scalars	1
1. Double addition	
2. Half double addition	
3. Exponentiation	
4. Complex numbers	
5. Affine transformations	
6. Matrix multiplication	
7. Modular multiplication	
8. Small operations	
9. Identity elements	
10. Complex inverses	
11. Affine inverses	
12. Matrix inverses	
13. Abelian groups	
14. Groups	
15. Independent group axioms	
16. Fields	
17. Addition and multiplication in fields	
18. Distributive failure	
19. Finite fields	
Chapter 2. Vectors	17
20. Vector spaces	
21. Examples	
22. Linear combinations	
23. Subspaces	
24. Unions of subspaces	

25. Spans	
26. Equalities of spans	
27. Some special spans	
28. Sums of subspaces	
29. Distributive subspaces	
30. Total sets	
31. Dependence	
32. Independence	
Chapter 3. Bases	39
33. Exchanging bases	
34. Simultaneous complements	
35. Examples of independence	
36. Independence over $\mathbb{R}$ and $\mathbb{Q}$	
37. Independence in $\mathbb{C}^2$	
38. Vectors common to different bases	
39. Bases in $\mathbb{C}^3$	
40. Maximal independent sets	
41. Complex as real	
42. Subspaces of full dimension	
43. Extended bases	
44. Finite-dimensional subspaces	
45. Minimal total sets	
46. Existence of minimal total sets	
47. Infinitely total sets	
48. Relatively independent sets	
49. Number of bases in a finite vector space	
50. Direct sums	
51. Quotient spaces	
52. Dimension of a quotient space	
53. Additivity of dimension	
Chapter 4. Transformations	51
54. Linear transformations	
55. Domain and range	
56. Kernel	
57. Composition	
58. Range inclusion and factorization	
59. Transformations as vectors	
60. Invertibility	
61. Invertibility examples	
62. Determinants: 2×2	

- 63. Determinants: $n \times n$
- 64. Zero-one matrices
- 65. Invertible matrix bases
- 66. Finite-dimensional invertibility
- 67. Matrices
- 68. Diagonal matrices
- 69. Universal commutativity
- 70. Invariance
- 71. Invariant complements
- 72. Projections
- 73. Sums of projections
- 74. not quite idempotence

Chapter 5. Duality 85

- 75. Linear functionals
- 76. Dual spaces
- 77. Solution of equations
- 78. Reflexivity
- 79. Annihilators
- 80. Double annihilators
- 81. Adjoints
- 82. Adjoints of projections
- 83. Matrices of adjoints

Chapter 6. Similarity 97

- 84. Change of basis: vectors
- 85. Change of basis: coordinates
- 86. Similarity: transformations
- 87. Similarity: matrices
- 88. Inherited similarity
- 89. Similarity: real and complex
- 90. Rank and nullity
- 91. Similarity and rank
- 92. Similarity of transposes
- 93. Ranks of sums
- 94. Ranks of products
- 95. Nullities of sums and products
- 96. Some similarities
- 97. Equivalence
- 98. Rank and equivalence

Chapter 7. Canonical Forms	107
99. Eigenvalues	
100. Sums and products of eigenvalues	
101. Eigenvalues of products	
102. Polynomials in eigenvalues	
103. Diagonalizing permutations	
104. Polynomials in eigenvalues, converse	
105. Multiplicities	
106. Distinct eigenvalues	
107. Comparison of multiplicities	
108. Triangularization	
109. Complexification	
110. Unipotent transformation	
111. Nilpotence	
112. Nilpotent products	
113. Nilpotent direct sums	
114. Jordan form	
115. Minimal polynomials	
116. Non-commutative Lagrange interpolation	
Chapter 8. Inner Product Spaces	129
117. Inner products	
118. Polarization	
119. The Pythagorean theorem	
120. The parallelogram law	
121. Complete orthonormal sets	
122. Schwarz inequality	
123. Orthogonal complements	
124. More linear functionals	
125. Adjoints on inner product spaces	
126. Quadratic forms	
127. Vanishing quadratic forms	
128. Hermitian transformations	
129. Skew transformations	
130. Real Hermitian forms	
131. Positive transformations	
132. positive inverses	
133. Perpendicular projections	
134. Projections on $\mathbb{C} \times \mathbb{C}$	
135. Projection order	
136. Orthogonal projections	

137. Hermitian eigenvalues	
138. Distinct eigenvalues	
Chapter 9. Normality	149
139. Unitary transformations	
140. Unitary matrices	
141. Unitary involutions	
142. Unitary triangles	
143. Hermitian diagonalization	
144. Square roots	
145. Polar decomposition	
146. Normal transformations	
147. Normal diagonalizability	
148. Normal commutativity	
149. Adjoint commutativity	
150. Adjoint intertwining	
151. Normal products	
152. Functions of transformations	
153. Gramians	
154. Monotone functions	
155. Reducing ranges and kernels	
156. Truncated shifts	
157. Non-positive square roots	
158. Similar normal transformations	
159. Unitary equivalence of transposes	
160. Unitary and orthogonal equivalence	
161. Null convergent powers	
162. Power boundedness	
163. Reduction and index 2	
164. Nilpotence and reduction	
Hints	169
Solutions:	
Chapter 1	185
Chapter 2	204
Chapter 3	216
Chapter 4	228
Chapter 5	252
Chapter 6	259
Chapter 7	277
Chapter 8	296
Chapter 9	310