
Contents

Preface	v
-------------------	---

CHAPTER I	Elementary results	1
-----------	-------------------------------------	----------

1	Definitions and notation	1
2	Annihilators	2
3	Annihilator series	4
4	Subdirectly irreducible nilpotent rings	7
5	The primary decomposition	9
6	The circle group	10

CHAPTER II	Examples of nilpotent rings	17
------------	--	-----------

1	Representations and rings of matrices	17
2	Free rings	19
3	The construction of rings by generators and relations	20
4	Modular group algebras of p -groups	24

CHAPTER III	Families and capability	27
-------------	--	-----------

1	The family classification	27
2	On the rings in a family	29
3	An example: the nilpotent rings of order p^3	34
4	Nilpotent rings with chain conditions	40
5	Capability; the construction of families	42
6	Conditions for capability	48

CHAPTER IV	The subring structure of nilpotent rings	51
------------	---	-----------

1	On generating rings	51
2	Automorphisms	54
3	Enumeration results	59
4	Nilpotent rings with only one subring of a given order	62

- 5 Nilpotent p -rings with one subring of order p 67
 6 Rings in which all subrings are ideals 68

CHAPTER V Counting finite nilpotent rings 80

- 1 Introduction 80
 2 Asymptotic results 81
 3 Asymptotic results for non-nilpotent rings 90

CHAPTER VI APPENDIX The construction of some special rings 95

- 1 Some nilpotent p -rings of rank 2 95
 2 The nilpotent algebras of dimensions 4 99
 3 Assume $\dim A = 4$, $A^3 \neq 0$ 101
 4 Assume $\dim A = 4$, $\dim A^2 = 1$, $A^3 = 0$,
 and A is not reducible 102
 5 Assume $\dim A = 4$, $\dim A^2 = 2$, $A^3 = 0$ 104

Bibliography 109

List of special notation 123

Index 125