Contents

Preface to the Third Edition xi
Software xiii
Selected References xv

1 Matrix Multiplication Problems 1

1.1 Basic Algorithms and Notation 2
1.2 Exploiting Structure 16
1.3 Block Matrices and Algorithms 24
1.4 Vectorization and Re-Use Issues 34

2 Matrix Analysis 48

2.1 Basic Ideas from Linear Algebra 48
2.2 Vector Norms 52
2.3 Matrix Norms 54
2.4 Finite Precision Matrix Computations 59
2.5 Orthogonality and the SVD 69
2.6 Projections and the CS Decomposition 75
2.7 The Sensitivity of Square Linear Systems 80

3 General Linear Systems 87

3.1 Triangular Systems 88
3.2 The LU Factorization 94
3.3 Roundoff Analysis of Gaussian Elimination 104
3.4 Pivoting 109
3.5 Improving and Estimating Accuracy 123

4 Special Linear Systems 133

4.1 The LDM^T and LDL^T Factorizations 135
4.2 Positive Definite Systems 140
4.3 Banded Systems 152
4.4 Symmetric Indefinite Systems 161
4.5 Block Systems 174
4.6 Vandermonde Systems and the FFT 183
4.7 Toeplitz and Related Systems 193

5 Orthogonalization and Least Squares 206

5.1 Householder and Givens Matrices 208
5.2 The QR Factorization 223
5.3 The Full Rank LS Problem 236
5.4 Other Orthogonal Factorizations 248
5.5 The Rank Deficient LS Problem 256
5.6 Weighting and Iterative Improvement 264
5.7 Square and Underdetermined Systems 270

6 Parallel Matrix Computations 275

6.1 Basic Concepts 276
6.2 Matrix Multiplication 292
6.3 Factorizations 300

7 The Unsymmetric Eigenvalue Problem 308

7.1 Properties and Decompositions 310
7.2 Perturbation Theory 320
7.3 Power Iterations 330
7.4 The Hessenberg and Real Schur Forms 341
7.5 The Practical QR Algorithm 352
7.6 Invariant Subspace Computations 362
7.7 The QZ Method for $Ax = \lambda Bx$ 375

8 The Symmetric Eigenvalue Problem 391

8.1 Properties and Decompositions 393
8.2 Power Iterations 405

8.3 The Symmetric QR Algorithm 414
8.4 Jacobi Methods 426
8.5 Tridiagonal Methods 439
8.6 Computing the SVD 448
8.7 Some Generalized Eigenvalue Problems 461

9 Lanczos Methods 470

9.1 Derivation and Convergence Properties 471
9.2 Practical Lanczos Procedures 479
9.3 Applications to $Ax = b$ and Least Squares 490
9.4 Arnoldi and Unsymmetric Lanczos 499

10 Iterative Methods for Linear Systems 508

10.1 The Standard Iterations 509
10.2 The Conjugate Gradient Method 520
10.3 Preconditioned Conjugate Gradients 532
10.4 Other Krylov Subspace Methods 544

11 Functions of Matrices 555

11.1 Eigenvalue Methods 556
11.2 Approximation Methods 562
11.3 The Matrix Exponential 572

12 Special Topics 579

12.1 Constrained Least Squares 580
12.2 Subset Selection Using the SVD 590
12.3 Total Least Squares 595
12.4 Computing Subspaces with the SVD 601
12.5 Updating Matrix Factorizations 606
12.6 Modified/Structured Eigenproblems 621

Bibliography 637
Index 687