

Contents

Foreword	v
Preface	ix
1 In the Ashes of the Ether: Differential Topology	1
§1. Smooth Manifolds	3
§2. Submanifolds	17
§3. Fiber Bundles	28
§4. Tangent Vectors, Bundles, and Fields	39
§5. Differential Forms	52
2 Looking for the Forest in the Leaves: Foliations	65
§1. Integral Curves	66
§2. Distributions	74
§3. Integrability Conditions	75
§4. The Frobenius Theorem	76
§5. The Frobenius Theorem in Terms of Differential Forms	79
§6. Foliations	81
§7. Leaf Holonomy	83
§8. Simple Foliations	90
3 The Fundamental Theorem of Calculus	95
§1. The Maurer–Cartan Form	96
§2. Lie Algebras	101

§3. Structural Equation	108
§4. Adjoint Action	110
§5. The Darboux Derivative	115
§6. The Fundamental Theorem: Local Version	116
§7. The Fundamental Theorem: Global Version	118
§8. Monodromy and Completeness	128
4 Shapes Fantastic: Klein Geometries	137
§1. Examples of Planar Klein Geometries	140
§2. Principal Bundles: Characterization and Reduction	144
§3. Klein Geometries	150
§4. A Fundamental Property	160
§5. The Tangent Bundle of a Klein Geometry	162
§6. The Meteor Tracking Problem	164
§7. The Gauge View of Klein Geometries	166
5 Shapes High Fantastical: Cartan Geometries	171
§1. The Base Definition of Cartan Geometries	173
§2. The Principal Bundle Hidden in a Cartan Geometry	178
§3. The Bundle Definition of a Cartan Geometry	184
§4. Development, Geometric Orientation, and Holonomy	202
§5. Flat Cartan Geometries and Uniformization	211
§6. Cartan Space Forms	217
§7. Symmetric Spaces	225
6 Riemannian Geometry	227
§1. The Model Euclidean Space	228
§2. Euclidean and Riemannian Geometry	234
§3. The Equivalence Problem for Riemannian Metrics	237
§4. Riemannian Space Forms	241
§5. Subgeometry of a Riemannian Geometry	244
§6. Isoparametric Submanifolds	259
7 Möbius Geometry	265
§1. The Möbius and Weyl Models	267
§2. Möbius and Weyl Geometries	277
§3. Equivalence Problems for a Conformal Metric	284
§4. Submanifolds of Möbius Geometry	294
§5. Immersed Curves	318
§6. Immersed Surfaces	323
8 Projective Geometry	331
§1. The Projective Model	332
§2. Projective Cartan Geometries	338
§3. The Geometry of Geodesics	343

§4. The Projective Connection in a Riemannian Geometry . .	349
§5. A Brief Tour of Projective Geometry	353
A Ehresmann Connections	357
§1. The Geometric Origin of Ehresmann Connections	358
§2. The Reductive Case	362
§3. Ehresmann Connections Generalize Cartan Connections .	365
§4. Covariant Derivative	371
B Rolling Without Slipping or Twisting	375
§1. Rolling Maps	376
§2. The Existence and Uniqueness of Rolling Maps	378
§3. Relation to Levi-Civita and Normal Connections	382
§4. Transitivity of Rolling Without Slipping or Twisting . . .	388
C Classification of One-Dimensional Effective Klein Pairs	391
§1. Classification of One-Dimensional Effective Klein Pairs . .	391
D Differential Operators Obtained from Symmetry	397
§1. Real Representations of $SO_2(\mathbf{R})$	397
§2. Operators on Riemannian Surfaces	400
E Characterization of Principal Bundles	407
Bibliography	411
Index	417