

Contents

Preface	ix
Nonlinear Equations	1
Lecture 1	3
By the dawn's early light	3
Interval bisection	4
Relative error	7
Lecture 2	9
Newton's method	9
Reciprocals and square roots	11
Local convergence analysis	12
Slow death	14
Lecture 3	17
A quasi-Newton method	17
Rates of convergence	20
Iterating for a fixed point	21
Multiple zeros	24
Ending with a proposition	25
Lecture 4	27
The secant method	27
Convergence	29
Rate of convergence	31
Multipoint methods	33
Muller's method	33
The linear-fractional method	34
Lecture 5	37
A hybrid method	37
Errors, accuracy, and condition numbers	40
Floating-Point Arithmetic	43
Lecture 6	45
Floating-point numbers	45
Overflow and underflow	47
Rounding error	48
Floating-point arithmetic	49
Lecture 7	53
Computing sums	53
Backward error analysis	55
Perturbation analysis	57
Cheap and chippy chopping	58
Lecture 8	61

Cancellation	61
The quadratic equation	61
That fatal bit of rounding error	63
Envoi	65

Linear Equations 67

Lecture 9	69
Matrices, vectors, and scalars	69
Operations with matrices	70
Rank-one matrices	73
Partitioned matrices	74
Lecture 10	77
The theory of linear systems	77
Computational generalities	78
Triangular systems	79
Operation counts	81
Lecture 11	83
Memory considerations	83
Row-oriented algorithms	83
A column-oriented algorithm	84
General observations on row and column orientation	86
Basic linear algebra subprograms	86
Lecture 12	89
Positive-definite matrices	89
The Cholesky decomposition	90
Economics	94
Lecture 13	97
Inner-product form of the Cholesky algorithm	97
Gaussian elimination	98
Lecture 14	103
Pivoting	103
BLAS	108
Upper Hessenberg and tridiagonal systems	110
Lecture 15	113
Vector norms	113
Matrix norms	114
Relative error	115
Sensitivity of linear systems	116
Lecture 16	119
The condition of a linear system	119
Artificial ill-conditioning	120
Rounding error and Gaussian elimination	122
Comments on the error analysis	125

Lecture 17	127
Introduction to a project	127
More on norms	127
The wonderful residual	128
Matrices with known condition numbers	129
Invert and multiply	130
Cramer's rule	130
Submission	131
Polynomial Interpolation	133
Lecture 18	135
Quadratic interpolation	135
Shifting	136
Polynomial interpolation	137
Lagrange polynomials and existence	137
Uniqueness	138
Lecture 19	141
Synthetic division	141
The Newton form of the interpolant	142
Evaluation	142
Existence and uniqueness	143
Divided differences	144
Lecture 20	147
Error in interpolation	147
Error bounds	149
Convergence	150
Chebyshev points	151
Numerical Integration	155
Lecture 21	157
Numerical integration	157
Change of intervals	158
The trapezoidal rule	158
The composite trapezoidal rule	160
Newton–Cotes formulas	161
Undetermined coefficients and Simpson's rule	162
Lecture 22	165
The Composite Simpson rule	165
Errors in Simpson's rule	166
Treatment of singularities	167
Gaussian quadrature: The idea	169
Lecture 23	171
Gaussian quadrature: The setting	171

Orthogonal polynomials	171
Existence	173
Zeros of orthogonal polynomials	174
Gaussian quadrature	175
Error and convergence	176
Examples	176
Numerical Differentiation	179
Lecture 24	181
Numerical differentiation and integration	181
Formulas from power series	182
Limitations	184
Bibliography	187
Introduction	187
References	187
Index	191