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**APPENDIX: A MODULAR SYSTEM
OF ALGORITHMS
FOR UNCONSTRAINED MINIMIZATION
AND NONLINEAR EQUATIONS**
(by Robert Schnabel)

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