

Contents

1	The Real Numbers	1
1.1	Preliminaries	1
1.2	Complete Ordered Fields	15
1.3	The Real Number System	24
1.4	Set Structures in $\mathbb{R}$	36
1.5	Normed Linear Spaces	47
2	Sequences in $\mathbb{R}$	55
2.1	Sequences and Convergence	55
2.2	Properties of Convergent Sequences	64
2.3	Completeness in $\mathbb{R}$ Revisited	74
2.4	Set Structures in $\mathbb{R}$ via Sequences	84
2.5	Complete Spaces	91
3	Numerical Series	99
3.1	Series of Real Numbers	99
3.2	Basic Convergence Tests	106
3.3	Absolute and Conditional Convergence	115
3.4	Sequence Spaces	127
4	Continuity	137
4.1	Sequences and the Limit of a Function	137
4.2	Continuity	148
4.3	The Intermediate Value Theorem	159
4.4	Continuity on a Set and Uniform Continuity	164
4.5	Bounded Linear Operators	172
5	The Derivative	183
5.1	The Definition of the Derivative	183
5.2	Properties of the Derivative	190
5.3	Value Theorems for the Derivative	197

5.4	Consequences of the Value Theorems	205
5.5	Taylor Polynomials	210
5.6	Eigenvalues and the Invariant Subspace Problem	216
6	Sequences and Series of Functions	227
6.1	Sequences of Functions	227
6.2	Series of Functions	241
6.3	Power Series	246
6.4	A Continuous Nowhere Differentiable Function	257
6.5	Spaces of Continuous Functions	263
7	The Riemann Integral	275
7.1	The Riemann Integral	275
7.2	Properties of the Riemann Integral	289
7.3	The Fundamental Theorem of Calculus	299
7.4	The Exponential Function	311
7.5	Spaces of Continuous Functions Revisited	317
8	Lebesgue Measure on $\mathbb{R}$	325
8.1	Length and Measure	325
8.2	Outer Measure on $\mathbb{R}$	334
8.3	Lebesgue Measure on $\mathbb{R}$	340
8.4	A Nonmeasurable Set	344
8.5	General Measure Theory	346
9	Lebesgue Integration	351
9.1	Measurable Functions	351
9.2	The Lebesgue Integral	360
9.3	Limits and the Lebesgue Integral	368
9.4	The L^p Spaces	383
	Epilogue	397
	References	403
	Index	405