

Contents

Preface	iii
Contributors	xvii
1 Markov Processes and Their Applications	1
<i>Rabi Bhattacharya</i>	
1.1 Introduction	1
1.2 Markov Chains.....	4
1.2.1 Simple Random Walk.....	5
1.2.2 Birth–Death Chains and the Ehrenfest Model	6
1.2.3 Galton–Watson Branching Process.....	7
1.2.4 Markov Chains in Continuous Time	8
1.2.5 References	11
1.3 Discrete Parameter Markov Processes on General State Spaces.....	11
1.3.1 Ergodicity of Harris Recurrent Processes.....	11
1.3.2 Iteration of I.I.D.Random Maps	14
1.3.3 Ergodicity of Non–Harris Processes.....	17
1.3.4 References	19
1.4 Continuous Time Markov Processes on General State Spaces	20
1.4.1 Processes with Independent Increments	20
1.4.2 Jump Processes.....	21
1.4.3 References	22
1.5 Markov Processes and Semigroup Theory	22
1.5.1 The Hille–Yosida Theorem	23
1.5.2 Semigroups and One-Dimensional Diffusions.....	25
1.5.3 References	31
1.6 Stochastic Differential Equations.....	31
1.6.1 Stochastic Integrals, SDE, Itô’s Lemma	32
1.6.2 Cameron–Martin–Girsanov Theorem and the Martingale Problem	35
1.6.3 Probabilistic Representation of Solutions to Elliptic and Parabolic Partial Differential Equations.....	38
1.6.4 References	39
Bibliography	41
2 Semimartingale Theory and Stochastic Calculus	47
<i>Jia-An Yan</i>	
2.1 General Theory of Stochastic Processes and Martingale Theory	48

2.1.1 Classical Theory of Martingales	48
2.1.2 General Theory of Stochastic Processes	52
2.1.3 Modern Martingale Theory	60
2.2 Stochastic Integrals	68
2.2.1 Stochastic Integrals w.r.t. Local Martingales	68
2.2.2 Stochastic Integrals w.r.t. Semimartingales	72
2.2.3 Convergence Theorems for Stochastic Integrals	75
2.2.4 Itô's Formula and Doléans Exponential Formula	78
2.2.5 Local Times of Semimartingales	81
2.2.6 Fisk-Stratonovich Integrals	82
2.2.7 Stochastic Differential Equations	84
2.3 Stochastic Calculus on Semimartingales	87
2.3.1 Stochastic Integration w.r.t. Random Measures	87
2.3.2 Characteristics of a Semimartingale	90
2.3.3 Processes with Independent Increments and Lévy Processes	91
2.3.4 Absolutely Continuous Changes of Probability	94
2.3.5 Martingale Representation Theorems	99
Bibliography	103
3 White Noise Theory	107
<i>Hui-Hsiung Kuo</i>	
3.1 Introduction	107
3.1.1 What is white noise?	107
3.1.2 White noise as the derivative of a Brownian motion	107
3.1.3 The use of white noise—a simple example	108
3.1.4 White noise as a generalized stochastic process	109
3.1.5 White noise as an infinite dimensional generalized function	110
3.2 White noise as a distribution theory	111
3.2.1 Finite dimensional Schwartz distribution theory	111
3.2.2 White noise space	112
3.2.3 Hida's original idea	112
3.2.4 Spaces of test and generalized functions	114
3.2.5 Examples of test and generalized functions	115
3.3 General spaces of test and generalized functions	117
3.3.1 Abstract white noise space	117
3.3.2 Wick tensors	118
3.3.3 Hida-Kubo-Takenaka space	119
3.3.4 Kondratiev-Streit space	120
3.3.5 Cochran-Kuo-Sengupta space	121
3.4 Continuous versions and analytic extensions	123
3.4.1 Continuous versions	123
3.4.2 Analytic extensions	125
3.4.3 Integrable functions	126
3.4.4 Generalized functions induced by measures	128
3.4.5 Generalized Radon-Nikodym derivative	129
3.5 Characterization theorems	131
3.5.1 The S -transform	131
3.5.2 Characterization of generalized functions	132
3.5.3 Convergence of generalized functions	136
3.5.4 Characterization of test functions	137
3.5.5 Intrinsic topology for the space of test functions	139

3.6 Continuous operators and adjoints.....	140
3.6.1 Differential operators.....	140
3.6.2 Translation and scaling operators.....	143
3.6.3 Multiplication and Wick product.....	144
3.6.4 Fourier-Gauss transform	146
3.6.5 Extensions to CKS-spaces.....	148
3.7 Comments on other topics and applications	150
Bibliography	155
4 SDEs and Their Applications	159
<i>Bo Zhang</i>	
4.1 SDEs with respect to Brownian motion	160
4.1.1 Ito type SDEs.....	160
4.1.2 Properties of solutions.....	163
4.1.3 Equations depending on a parameter	165
4.1.4 Stratonovich Stochastic Differential Equations.....	167
4.1.5 Stochastic Differential Equations on Manifolds.....	168
4.2 Applications.....	169
4.2.1 Diffusions	169
4.2.2 Boundary value problem	173
4.2.3 Optimal stopping	176
4.2.4 Stochastic control	180
4.2.5 Backward SDE and applications.....	185
4.3 Some generalizations of SDEs	191
4.3.1 SDEs of the jump type	191
4.3.2 SDE with respect to semimartingale	198
4.3.3 SDE driven by nonlinear integrator	204
4.4 Stochastic Functional Differential Equations	212
4.4.1 Existence and Uniqueness of Solution.....	212
4.4.2 Markov property	215
4.4.3 Regularity of the trajectory field	217
4.5 Stochastic Differential Equations in Abstract Spaces	219
4.5.1 Stochastic evolution equations	219
4.5.2 Dissipative stochastic systems	222
4.6 Anticipating Stochastic Differential Equation	224
4.6.1 Volterra equations with anticipating kernel	224
4.6.2 SDEs with anticipating drift and initial condition	227
Bibliography	229
5 Numerical Analysis of SDEs Without Tears	237
<i>H. Schurz</i>	
5.1 Introduction.....	237
5.2 The Standard Setting For (O)SDEs.....	238
5.3 Stochastic Taylor Expansions	241
5.3.1 The Itô Formula (Itô's Lemma)	241
5.3.2 The main idea of stochastic Itô's-Taylor expansions.....	241
5.3.3 Hierarchical sets, coefficient functions, multiple integrals	243
5.3.4 Amore compact formulation.....	243
5.3.5 The example of Geometric Brownian Motion	244
5.3.6 Key relations between multiple integrals.....	245
5.4 A Toolbox of Numerical Methods.....	246

5.4.1 The explicit and fully drift-implicit Euler method	246
5.4.2 The family of stochastic Theta methods	247
5.4.3 Trapezoidal and midpoint methods	248
5.4.4 Rosenbrock methods (RTMs)	248
5.4.5 Balanced implicit methods (BIMs)	249
5.4.6 Predictor-corrector methods (PCMs)	249
5.4.7 Explicit Runge-Kutta methods (RKMs)	250
5.4.8 Newton's method	251
5.4.9 The explicit and implicit Mil'shtein methods	252
5.4.10 Gaines's representation of Mil'shtein method	253
5.4.11 Generalized Theta-Platen methods	254
5.4.12 Talay-Tubaro extrapolation technique and linear PDEs	254
5.4.13 Denk-Hersch method for highly oscillating systems	255
5.4.14 Stochastic Adams-type methods	257
5.4.15 The two step Mil'shtein method of Horváth-Bokor	258
5.4.16 Higher order Taylor methods	258
5.4.17 Splitting methods of Petersen-Schurz	258
5.4.18 The ODE method with commutative noise	260
5.4.19 Random local linearization methods (LLM)	262
5.4.20 Simultaneous time and chance discretizations	264
5.4.21 Stochastic waveform relaxation methods	264
5.4.22 Comments on numerical analysis of SPDEs	264
5.4.23 General concluding comment on numerical methods	265
5.5 On the Main Principles of Numerics	265
5.5.1 ID-invariance	265
5.5.2 Numerical p th mean consistency	266
5.5.3 Numerical p th mean stability	266
5.5.4 Numerical p th mean contractivity	267
5.5.5 Numerical p th mean convergence	267
5.5.6 The main principle: combining all concepts from 5.1-5.5	268
5.5.7 On fundamental crossrelations	273
5.6 Results on Convergence Analysis	276
5.6.1 Continuous time convergence concepts	276
5.6.2 On key relations between convergence concepts	278
5.6.3 Fundamental theorems of mean square convergence	278
5.6.4 Strong mean square convergence theorem	280
5.6.5 The Clark-Cameron mean square order bound in $\mathbb{R}^1$	280
5.6.6 Exact mean square order bounds of Cambanis and Hu	282
5.6.7 A theorem on double L^2 -convergence with adaptive Δ_n	284
5.6.8 The fundamental theorem of weak convergence	285
5.6.9 Approximation of some functionals	286
5.6.10 The pathwise error process for explicit Euler methods	289
5.6.11 Almost sure convergence	289
5.7 Numerical Stability, Stationarity, Boundedness, and Invariance	291
5.7.1 Stability of linear systems with multiplicative noise	291
5.7.2 Stationarity of linear systems with additive noise	294
5.7.3 Asymptotically exact methods for linear systems	296
5.7.4 Almost sure nonnegativity of numerical methods	297
5.7.5 Numerical invariance of intervals $[0, M]$	299
5.7.6 Preservation of boundaries for Brownian Bridges	301
5.7.7 Nonlinear stability of implicit Euler methods	302

5.7.8 Linear and nonlinear A-stability	303
5.7.9 Stability exponents of explicit-implicit methods	304
5.7.10 Hofmann-Platen's M-stability concept in $\mathbb{C}^1$	306
5.7.11 Asymptotic stability with probability one	308
5.8 Numerical Contractivity	309
5.8.1 Contractivity of SDEs with monotone coefficients	309
5.8.2 Contractivity of implicit Euler methods	310
5.8.3 p th mean B- and BN-stability	310
5.8.4 Contractivity exponents of explicit-implicit methods	311
5.8.5 General V-asymptotics of discrete time iterations	312
5.8.6 An example for discrete time V-asymptotics	314
5.8.7 Asymptotic contractivity with probability one	317
5.9 On Practical Implementation	317
5.9.1 Implementation issues: some challenging examples	317
5.9.2 Generation of pseudorandom numbers	321
5.9.3 Substitutions of randomness under weak convergence	323
5.9.4 Are quasi random numbers useful for (O)SDEs	324
5.9.5 Variable step size algorithms	325
5.9.6 Variance reduction techniques	326
5.9.7 How to estimate p th mean errors	328
5.9.8 On software and programmed packages	329
5.9.9 Comments on applications of numerics for (O)SDEs	329
5.10 Comments, Outlook, Further Developments	330
5.10.1 Recent and further developments	330
5.10.2 General comments	330
5.10.3 Acknowledgements	331
5.10.4 New trends -10 challenging problem areas	331
Bibliography	333

6 Large Deviations and Applications 361

Amir Dembo and Ofer Zeitouni

6.1 Introduction	361
6.2 The Large Deviation Principle	363
6.3 Large Deviation Principles for Finite Dimensional Spaces	365
6.3.1 The Method of Types	366
6.3.2 Cramér's Theorem in $\mathbb{R}^d$	368
6.3.3 The Gärtner-Ellis Theorem	369
6.3.4 Inequalities for Bounded Martingale Differences	371
6.3.5 Moderate Deviations and Exact Asymptotics	372
6.4 General Properties	373
6.4.1 Existence of an LDP and Related Properties	374
6.4.2 Contraction Principles and Exponential Approximation	376
6.4.3 Varadhan's Lemma and its Converse	380
6.4.4 Convexity Considerations	382
6.4.5 Large Deviations for Projective Limits	385
6.5 Sample Path LDPs	388
6.5.1 Sample Path Large Deviations for Random Walk and for Brownian Motion	388
6.5.2 The Freidlin-Wentzell Theory	390
6.5.3 Application: The Problem of Diffusion Exit from a Domain	392
6.6 LDPs for Empirical Measures	396

6.6.1 Cramér's Theorem in Polish Spaces	396
6.6.2 Sanov's Theorem.....	399
6.6.3 LDP for Empirical Measures of Markov Chains	401
6.6.4 Mixing Conditions and LDP	404
6.6.5 Application: The Gibbs Conditioning Principle	406
6.6.6 Application: The Hypothesis Testing Problem	410
Bibliography	413
7 Stability and Stabilizing Control of Stochastic Systems	417
<i>P. V. Pakshin</i>	
7.1 Stochastic mathematical models of systems	419
7.1.1 Models of differential systems corrupted by noise	419
7.1.2 Models of differential systems with random jumps	422
7.1.3 Differential generator	423
7.2 Stochastic control problem	425
7.2.1 Preliminaries	425
7.2.2 Stochastic dynamic programming	426
7.2.3 Stochastic maximum principle	430
7.2.4 Separation principle	432
7.3 Definition of stochastic stability and stochastic Lyapunov function.....	438
7.3.1 Classic stability concept.....	438
7.3.2 Weak Lyapunov stability	438
7.3.3 Strong Lyapunov stability	439
7.3.4 Mean square and p -stability	439
7.3.5 Recurrence and positivity	440
7.3.6 Stochastic Lyapunov function	441
7.4 General stability and stabilization theorems	442
7.4.1 Stability in probability theorems	442
7.4.2 Recurrence and positivity theorems.....	442
7.4.3 p th mean stability theorems and their inversion.....	443
7.4.4 Stability in the first order approximation	446
7.4.5 Stabilization problem and fundamental theorem	447
7.5 Instability	448
7.5.1 Classic stochastic instability concept.....	448
7.5.2 Nonpositivity and nonrecurrence	450
7.6 Stability criteria and testable conditions	451
7.6.1 General stability tests for linear systems.....	451
7.6.2 Some particular stability criteria for linear systems.....	452
7.6.3 Stability of the p th moments of linear systems.....	454
7.6.4 Absolute stochastic stability	455
7.6.5 Robust stability.....	456
7.7 Stabilizing control of linear system	458
7.7.1 General linear systems	458
7.7.2 Linear systems with parametric noise.....	459
7.7.3 Robust stabilizing control	464
Bibliography	467
8 Stochastic Differential Games and Applications	473
<i>K. M. Ramachandran</i>	
8.1 Introduction.....	473
8.2 Two person zero-sum differential games	475

8.2.1 Two person zero-sum games: martingale methods	475
8.2.2 Two person zero-sum games and viscosity solutions	484
8.2.3 Stochastic differential games with multiple modes	487
8.3 <i>N</i> -Person stochastic differential games	490
8.3.1 Discounted payoff on the infinite horizon	491
8.3.2 Ergodic payoff	492
8.4 Weak convergence methods in differential games	498
8.4.1 Weak convergence preliminaries	498
8.4.2 Weak convergence in <i>N</i> -person stochastic differential games	500
8.4.3 Partially observed stochastic differential games and weak convergence	510
8.5 Applications	518
8.5.1 Stochastic equity investment model with institutional investor speculation	519
8.6 Conclusion	523
Bibliography	525
9 Stochastic Manufacturing Systems: A Hierarchical Control Approach	533
<i>Q. Zhang</i>	
9.1 Introduction	533
9.2 Single Machine System	535
9.3 Flowshops	538
9.4 Jobshops	541
9.5 Production-Capacity Expansion Models	542
9.6 Production-Marketing Models	548
9.7 Risk-Sensitive Control	550
9.8 Optimal Control	553
9.9 Hierarchical Control	555
9.10 Risk-Sensitive Control	557
9.11 Constant Product Demand	560
9.12 Constant Machine Capacity	566
9.13 Marketing-Production with a Jump Demand	568
9.14 Concluding Remarks	571
Bibliography	573
10 Stochastic Approximation: Theory and Applications	577
<i>G. Yin</i>	
10.1 Introduction	577
10.1.1 Historical Development	578
10.1.2 Basic Issues	579
10.1.3 Outline of the Chapter	579
10.2 Algorithms and Variants	579
10.2.1 Basic Algorithm	579
10.2.2 More General Algorithms	581
10.2.3 Projection and Truncation Algorithms	582
10.2.4 Global Stochastic Approximation	584
10.2.5 Continuous-time Stochastic Approximation Algorithms	585
10.2.6 Stochastic Approximation in Function Spaces	585
10.3 Convergence	585
10.3.1 ODE Methods	586
10.3.2 Weak Convergence Method	588

10.4 Rates of Convergence.....	590
10.4.1 Scaling Factor α	590
10.4.2 Tightness of the Scaled Estimation Error	591
10.4.3 Local Analysis.....	592
10.4.4 Random Directions.....	594
10.4.5 Stopping Rules.....	594
10.5 Large Deviations.....	594
10.5.1 Motivation.....	595
10.5.2 Large Deviations for Stochastic Approximation	595
10.6 Asymptotic Efficiency	596
10.6.1 Iterate Averaging	597
10.6.2 Smoothed Algorithms.....	598
10.6.3 Some Numerical Data	600
10.7 Applications.....	601
10.7.1 Adaptive Filtering	602
10.7.2 Adaptive Beam Forming	602
10.7.3 System Identification and Adaptive Control	603
10.7.4 Adaptive Step-size Tracking Algorithms	605
10.7.5 Approximation of Threshold Control Policies.....	606
10.7.6 GI/G/1 Queue	607
10.7.7 Distributed Algorithms for Supervised Learning	608
10.7.8 A Heat Exchanger	610
10.7.9 Evolutionary Algorithms	612
10.7.10 Digital Diffusion Machines	613
10.8 Further Remarks	614
10.8.1 Convergence.....	614
10.8.2 Rate of Convergence.....	615
10.8.3 Law of Iterated Logarithms.....	615
10.8.4 Robustness	616
10.8.5 Parallel Stochastic Approximation.....	616
10.8.6 Open Questions	617
10.8.7 Conclusion.....	617
Bibliography.....	619
11 Optimization by Stochastic Methods	625
<i>Franklin Mendivil, R. Shonkwiler, and M.C. Spruill</i>	
11.1 Nature of the problem	625
11.1.1 Introduction	625
11.1.2 No Free Lunch	626
11.1.3 The Permanent Problem	628
11.2 A Brief Survey of Some Methods for Global Optimization	629
11.2.1 Covering Methods.....	630
11.2.2 Branch and bound	631
11.2.3 Iterative Improvement.....	632
11.2.4 Trajectory/tunneling Methods	633
11.2.5 Tabu search.....	634
11.2.6 Random Search.....	634
11.2.7 Multistart.....	635
11.3 Markov Chain and Renewal Theory Considerations.....	635
11.3.1 IIP parallel search.....	638
11.3.2 Restarted Improvement Algorithms	639

11.3.3	Renewal Techniques in Restarting	642
11.4	Simulated Annealing	644
11.4.1	Introduction	644
11.4.2	Simulated annealing applied to the permanent problem	646
11.4.3	Convergence Properties of Simulated Annealing and Related Algorithms	647
11.5	Restarted Algorithms	653
11.5.1	Introduction	653
11.5.2	The Permanent Problem using restarted simulated annealing	654
11.5.3	Restarted Simulated Annealing	655
11.5.4	Numerical comparisons	656
11.6	Evolutionary Computations	658
11.6.1	Introduction	658
11.6.2	A GA for the permanent problem	660
11.6.3	Some specific Algorithms	661
11.6.4	GA principles, schemata, multi-armed bandit, implicit parallelism	662
11.6.5	A genetic algorithm for constrained optimization problems	667
11.6.6	Markov Chain Analysis Particular to Genetic Algorithms	670
	Bibliography	673
12	Stochastic Control Methods in Asset Pricing	679
	<i>Thaleia Zariphopoulou</i>	
12.1	Introduction	679
12.2	The Hamilton-Jacobi-Bellman (HJB) equation	680
12.3	Models of Optimal Investment and Consumption I	684
12.3.1	Merton models with intermediate consumption	687
12.3.2	Merton models with non-linear stock dynamics	689
12.3.3	Merton models with trading constraints	691
12.3.4	Merton models with non-homogeneous investment opportunities	693
12.3.5	Models of Optimal Portfolio Management with General Utilities	699
12.3.6	Optimal goal problems	703
12.3.7	Alternative models of expected utility	705
12.4	Models of optimal investment and consumption II	707
12.4.1	Optimal investment/consumption models with transaction costs	707
12.4.2	Optimal investment/consumption models with stochastic labor income	719
12.5	Expected utility methods in derivative pricing	723
12.5.1	The Black and Scholes valuation formula	725
12.5.2	Super-replicating strategies	727
12.5.3	The utility maximization theory	729
12.5.4	Imperfect hedging strategies	738
12.5.5	Other models of derivative pricing with transaction costs	742
	Bibliography	745
	Index	754