

Acknowledgement	ix
Chapter 1. Introduction	1
Chapter 2. Background	7
2.1. (Riemannian) laminations and Laplacians	7
2.2. Covering laminations	9
2.3. Heat kernels, (weakly) harmonic measures and Standing Hypotheses	10
2.4. Brownian motion and Wiener measures without holonomy	13
2.5. Wiener measures with holonomy	15
Chapter 3. Statement of the main results	19
3.1. Multiplicative cocycles	19
3.2. First Main Theorem and applications	21
3.3. Second Main Theorem and applications	24
3.4. Plan of the proof	26
Chapter 4. Preparatory results	29
4.1. Measurability issue	29
4.2. Markov property of Brownian motion	31
Chapter 5. Leafwise Lyapunov exponents	33
Chapter 6. Splitting subbundles	41
Chapter 7. Lyapunov forward filtrations	47
7.1. Oseledec type theorems	47
7.2. Fibered laminations and totally invariant sets	50
7.3. Cylinder laminations and end of the proof	57
Chapter 8. Lyapunov backward filtrations	67
8.1. Extended sample-path spaces	67
8.2. Leafwise Lyapunov backward exponents and Oseledec backward type theorem	80
Chapter 9. Proof of the main results	89
9.1. Canonical cocycles and specializations	89
9.2. $\mathcal{A}$ -weakly harmonic measures and splitting invariant bundles	91
9.3. First Main Theorem and Ledrappier type characterization of Lyapunov spectrum	106
9.4. Second Main Theorem and its corollaries	112

APPENDICES	118
Appendix A. Measure theory for sample-path spaces	119
A.1. Multifunctions and measurable selections	119
A.2. σ -algebras: approximations and measurability	121
A.3. σ -algebra $\mathcal{A}$ on a leaf	124
A.4. Holonomy maps	130
A.5. Metrizability and separability of sample-path spaces	131
A.6. The leafwise diagonal is Borel measurable	135
A.7. σ -algebra $\mathcal{A}$ on a lamination	135
A.8. The cylinder laminations are Riemannian continuous-like	142
A.9. The extended sample-path space is of full outer measure	143
Appendix B. Harmonic measure theory and ergodic theory for sample-path spaces	147
B.1. Fibered laminations	147
B.2. Relation between a lamination and its covering lamination	159
B.3. Invariance of Wiener measures with harmonic initial distribution	159
B.4. Ergodicity of Wiener measures with ergodic harmonic initial distribution	162
Bibliography	165
Index	167
Glossary of Notation	171