
Contents

Preface to the Series	xi
Preface to Part 1	xvii
Chapter 1. Preliminaries	1
§1.1. Notation and Terminology	1
§1.2. Metric Spaces	3
§1.3. The Real Numbers	6
§1.4. Orders	9
§1.5. The Axiom of Choice and Zorn's Lemma	11
§1.6. Countability	14
§1.7. Some Linear Algebra	18
§1.8. Some Calculus	30
Chapter 2. Topological Spaces	35
§2.1. Lots of Definitions	37
§2.2. Countability and Separation Properties	51
§2.3. Compact Spaces	63
§2.4. The Weierstrass Approximation Theorem and Bernstein Polynomials	76
§2.5. The Stone–Weierstrass Theorem	88
§2.6. Nets	93
§2.7. Product Topologies and Tychonoff's Theorem	99
§2.8. Quotient Topologies	103

Chapter 3. A First Look at Hilbert Spaces and Fourier Series	107
§3.1. Basic Inequalities	109
§3.2. Convex Sets, Minima, and Orthogonal Complements	119
§3.3. Dual Spaces and the Riesz Representation Theorem	122
§3.4. Orthonormal Bases, Abstract Fourier Expansions, and Gram–Schmidt	131
§3.5. Classical Fourier Series	137
§3.6. The Weak Topology	168
§3.7. A First Look at Operators	174
§3.8. Direct Sums and Tensor Products of Hilbert Spaces	176
Chapter 4. Measure Theory	185
§4.1. Riemann–Stieltjes Integrals	187
§4.2. The Cantor Set, Function, and Measure	198
§4.3. Bad Sets and Good Sets	205
§4.4. Positive Functionals and Measures via $L^1(X)$	212
§4.5. The Riesz–Markov Theorem	233
§4.6. Convergence Theorems; L^p Spaces	240
§4.7. Comparison of Measures	252
§4.8. Duality for Banach Lattices; Hahn and Jordan Decomposition	259
§4.9. Duality for L^p	270
§4.10. Measures on Locally Compact and σ -Compact Spaces	275
§4.11. Product Measures and Fubini’s Theorem	281
§4.12. Infinite Product Measures and Gaussian Processes	292
§4.13. General Measure Theory	300
§4.14. Measures on Polish Spaces	306
§4.15. Another Look at Functions of Bounded Variation	314
§4.16. Bonus Section: Brownian Motion	319
§4.17. Bonus Section: The Hausdorff Moment Problem	329
§4.18. Bonus Section: Integration of Banach Space-Valued Functions	337
§4.19. Bonus Section: Haar Measure on σ -Compact Groups	342

Chapter 5. Convexity and Banach Spaces	355
§5.1. Some Preliminaries	357
§5.2. Hölder's and Minkowski's Inequalities: A Lightning Look	367
§5.3. Convex Functions and Inequalities	373
§5.4. The Baire Category Theorem and Applications	394
§5.5. The Hahn–Banach Theorem	414
§5.6. Bonus Section: The Hamburger Moment Problem	428
§5.7. Weak Topologies and Locally Convex Spaces	436
§5.8. The Banach–Alaoglu Theorem	446
§5.9. Bonus Section: Minimizers in Potential Theory	447
§5.10. Separating Hyperplane Theorems	454
§5.11. The Krein–Milman Theorem	458
§5.12. Bonus Section: Fixed Point Theorems and Applications	468
Chapter 6. Tempered Distributions and the Fourier Transform	493
§6.1. Countably Normed and Fréchet Spaces	496
§6.2. Schwartz Space and Tempered Distributions	502
§6.3. Periodic Distributions	520
§6.4. Hermite Expansions	523
§6.5. The Fourier Transform and Its Basic Properties	540
§6.6. More Properties of Fourier Transform	548
§6.7. Bonus Section: Riesz Products	576
§6.8. Fourier Transforms of Powers and Uniqueness of Minimizers in Potential Theory	583
§6.9. Constant Coefficient Partial Differential Equations	588
Chapter 7. Bonus Chapter: Probability Basics	615
§7.1. The Language of Probability	617
§7.2. Borel–Cantelli Lemmas and the Laws of Large Numbers and of the Iterated Logarithm	632
§7.3. Characteristic Functions and the Central Limit Theorem	648
§7.4. Poisson Limits and Processes	660
§7.5. Markov Chains	667

Chapter 8. Bonus Chapter: Hausdorff Measure and Dimension	679
§8.1. The Carathéodory Construction	680
§8.2. Hausdorff Measure and Dimension	687
Chapter 9. Bonus Chapter: Inductive Limits and Ordinary Distributions	705
§9.1. Strict Inductive Limits	706
§9.2. Ordinary Distributions and Other Examples of Strict Inductive Limits	711
Bibliography	713
Symbol Index	765
Subject Index	769
Author Index	779
Index of Capsule Biographies	789