

ANNALES ACADEMIÆ SCIENTIARUM FENNICÆ

MATHEMATICA

DISSERTATIONES

150

ON THE MEAN SQUARE OF QUADRATIC
DIRICHLET L -FUNCTIONS AT 1

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