

Kelei Wang

Free Boundary
Problems and
Asymptotic Behavior
of Singularly
Perturbed Partial
Differential Equations

Contents

1	Introduction	1
1.1	Competing Species System	1
1.2	Bose–Einstein Condensates	6
1.3	Main Results	8
2	Uniqueness, Stability and Uniform Lipschitz Estimates	17
2.1	A Uniqueness Result for the Elliptic System	17
2.2	Asymptotics in the Parabolic Case	21
2.3	A Uniform Lipschitz Estimate	24
3	Uniqueness in the Singular Limit	27
3.1	An Interior Measure Estimate	27
3.2	Uniqueness in the Singular Limit	33
3.2.1	Verification of the Convergence of the Integration	40
3.3	Uniqueness and Asymptotics for the Singular Parabolic System	41
4	The Dynamics of One Dimensional Singular Limiting Problem	45
4.1	The Stationary Case	45
4.2	The Parabolic Case	48
5	Approximate Clean Up Lemma	61
5.1	Systems with Zeroth Order Terms	61
5.2	Proof of the Approximate Clean Up Lemma	66
5.3	A Linearized Version	72
5.4	A Boundary Version	78
6	Asymptotics in Strong Competition	81
6.1	Non-degeneracy Conditions and the Main Result	81
6.2	A Coarse Convergence Result	84
6.3	The Refined Convergence Result	89

7 The Limit Equation of a Singularly Perturbed System 95
 7.1 The Elliptic Case 95
 7.2 The Parabolic Case 99
References 107
Index 109