

CHEMICAL KINETICS

EDITED BY

R.G. COMPTON

M.A., D. Phil. (Oxon.)

*Oxford University
The Physical Chemistry Laboratory
Oxford, England*

G. HANCOCK

*Oxford University
The Physical Chemistry Laboratory
Oxford, England*

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MODERN ASPECTS OF DIFFUSION-CONTROLLED REACTIONS COOPERATIVE PHENOMENA IN BIMOLECULAR PROCESSES

E. KOTOMIN

V. KUZOVKOV

University of Latvia



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